Hanstein's Constructive Drawing

Geometrical Constructions.

CONSTRUCTIVE DRAWING.

A TEXT-BOOK FOR HOME INSTRUCTION, HIGH SCHOOLS, MANUAL TRAINING SCHOOLS, TECHNICAL SCHOOLS AND UNIVERSITIES.

ARRANGED BY

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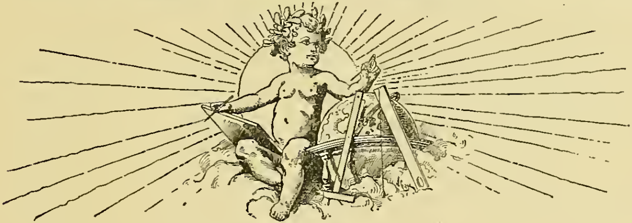

SECOND EDITION.

GEOMETRIC CONSTRUCTIONS.

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PREFACE

AT the request of assistants and pupils, as a help and for home instruction, I have compiled this course of constructive drawing, as it has been taught for the past twenty years in the Chicago City High-Schools, in the Drawing Department of the Chicago Mechanics' Institute and lately in the Columbian Trade and Business School.

A practical experience of seventeen years in office and shop and his occupation as teacher during the past twenty years have given the author such experience and judgment as to select only such problems as are of practical importance to those who follow architectural, mechanical and engineering vocations as well as problems which are indispensable to manufacturing and industrial pursuits.

As draughtsman, I have endeavored to arrange this work so as to bring into immediate application all the tools which are required in every drafting-room, and time and expense have not been spared to impress the student with the cardinal virtues of a successful draughtsman: *accuracy and cleanliness.*

The author will feel well rewarded for his trouble if men of ability and learning will think it worth their while to point out to him any deficiencies that they may notice in perusing this work.

HERMAN HANSTEIN

Chicago, Ill., September, 1894.

Preface to Second Edition

THIS editition has been revised and for the convenience of both student and teacher the number of constructions on each plate has been reduced one half.

In thanking colleagues for the kindness with which the first edition was received, the author hopes that in this new arrangement the "Geometric Constructions" will retain the favor of those who have followed this course in the preparation of Projection and Descriptive Geometry.

HERMAN HANSTEIN.

Chicago, Ill., June, 1904.

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Necessary Tools, Implements and Their Application.

Fig. 1, Plate 7.—*A drawing-board*, made of well-seasoned white pine, poplar (whitewood) or basswood, the lightest of our woods, answers this purpose best, as these woods are evenly grained and do not offer great obstruction to thumb-tacks, by which the drawing-paper is fastened to the board.

The under surface of this board should be provided with two parallel dovetailed grooves, 3 or 4 inches from edges O and O′ and right-angled to the grain of the wood, to receive not too *tightly* fitting cleats, at which the board may shrink, to prevent its splitting. The cleats therefore should not be glued in the grooves to receive them.

When one draws with the right hand, the straight edge, called T square (T), and triangle S, called set square, are operated with the left hand, and when one draws with the left hand the set and T square are operated with the right hand.

The T *is used only on one edge of the board.*

Figs. 1 and 2, Plate 3.—*Set squares* (Triangles).—One set square of 30° and 60° and one of 45° (degrees) are required, as shown in Plate 3 and these should be tested for accuracy before admitted to practical use.

Test.—Place the set square with one right-angle side to the T, as shown in Fig. 1, and draw with a hard (4 H) well-pointed lead pencil a line on side a b. Reverse the set square on a b as an axis, and if the line drawn and the side of the set square coincide (fall into one) the angle is a *right angle*, while a convergence will show the angle to be incorrect, and such a set square should not be used until it is made *true*. A similar test should also be made with the T square before using it.

Figs. 3 and 4.—Plate 3.—Figs. 3 and 4 show the different angles possible to be drawn with the assistance of both set squares and the T.

Fig. 2, Plate 1.—*The protractor* is a semi-circular instrument made of brass or celluloid. Point C represents the center of the semicircle, which is divided by radii into 180 equal parts. In measuring an angle, such as the angle B C A, place the instrument with its center at the vertex (the intersection of the sides of the angle), and one side to coincide with the diameter of the instrument. Note the number of divisions on the intervening arc, which is 137 (read 137° degrees); 1° = 60 m. (minutes) and 1 m. = 60 sec. (seconds).

THE SET OF DRAWING INSTRUMENTS.

FIGS. 4 to 7, PLATE 1.—*The very best is none too good.* A *set* should contain one pair of compasses, Fig. 4, with needle-point center, Fig. 4 D, a lead pencil attachment, Fig. 4 B, a ruling-pen for circles, Fig. 4 A, one pair of dividers, Fig. 5, and one or two straight ruling-pens, Fig. 6, of different sizes. For boilermakers, machinists, architectural iron constructors, etc., a set of *bow* instruments is a valuable addition to the above.

FIGS. 7 and 7 A, PLATE 1.—*The lead* (6 H) for the compasses is bought in sticks of 5 in. in length and $\frac{1}{16}$ in. thick. Take a length $\frac{1}{4}$ in. longer than the length of the hole in the attachment to receive it. Give the lead the shape shown in Fig. 7, which is most conveniently done on a piece of emery paper or a fine file; then take off the corners as indicated by the lines G K and H I Fig. 7 A. Insert it with a flat side towards the center of the compasses and clamp it tight with the clampscrew S, Fig. 4 B.

The main joint near the handle ought to move with ease, and one hand should be sufficient to open or close dividers or compasses easily.

The straight pen and the pen for circular ruling must be treated most carefully. Their blades are of the same length, not so pointed and sharp as to cut the paper, and when filled with ink should be entirely free of ink on the outside.

In inking circles, the legs of the compasses should be bent at the joints P and O (Fig. 4) sufficiently to have both blades touch paper equally to allow an even flow of the ink. The leg which carries the center of the compasses should have a vertical position so as to avoid a tapering of the hole in the paper by its revolution.

The correct position of the compasses is shown in Fig. 4, where the line M N represents the surface of the drawing-paper. A convenient arrangement to keep the plates of the course for later reference is used in the Chicago High Schools. The sheets of paper of 11 in. × 17 in. are perforated and seamed by laces in a portefolio of 12 in. × 18 in. A rectangle as a border line of 10 in. × 15 in. encloses the drawing surface which is divided into six equal squares 5 in. on a side, each to receive one construction. See Plate 4. Larger spaces however should be used to execute accurately some of the constructions, for which the proportional sizes may be ascertained from the corresponding plates. Draw the lines light and carefully with Dixon's V H (very hard), Faber 4 H (Siberian), or a Hartmuth 6 H (compressed lead) pencil, having a fine round point.

Inking the drawing.—Execute all constructions in pencil, to admit of corrections, when necessary, before inking them. It is also advisable for the inexperienced to *write* the required text on the drawing in pencil, to distribute letters and words regularly in the available space beneath each drawing, as shown in Fig. 1, Plate 4, before writing with *Indian ink.*

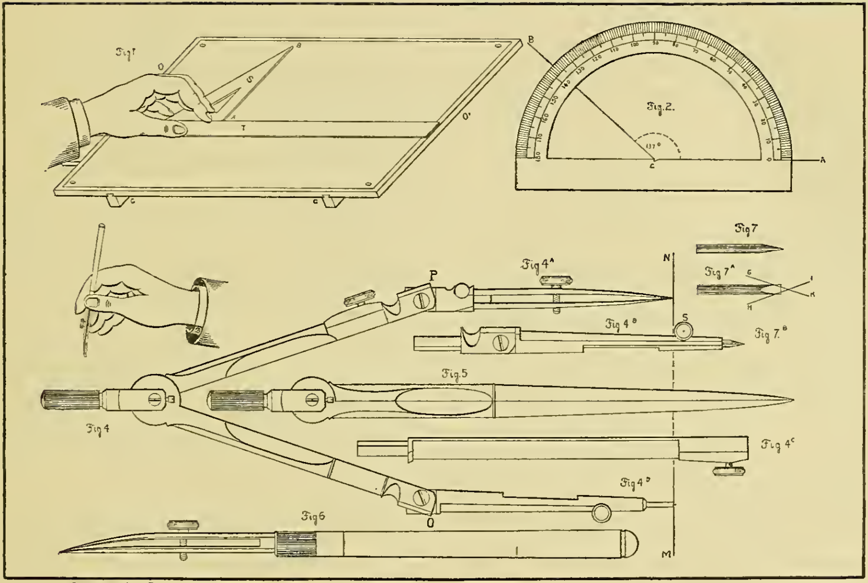


Fig 1
Fig. 2.
Fig 4
Fig 4^A
Fig 4^B
Fig 4^C
Fig 4^D
Fig. 5
Fig 6
Fig 7
Fig 7^A
Fig 7.^B

PLATES 2 and 3.—*Alphabets.*—Several styles of lettering for titles of drawings commonly used, are shown on Plates 2 and 3.

PLATE 3—In Fig. 5, A B C D E F G and H show a few samples of corners in border lines for elaborate work.

The following distinctions of inked lines in drawing are made to recognize readily all that pertains to *problem*, *construction* and *result*.

THE PROBLEM LINE *is drawn fine and uninterrupted.*

THE CONSTRUCTION LINE *is fine and dashed.*

THE RESULT, *a strong, uninterrupted line.*

Begin inking with construction arcs and circles, then the circular problem lines, and then the circular result lines.

This is done so as to save time, to avoid the change of tool in hand, and not to clean and re-set the pen ofteuer than necessary.

Construction straight lines are drawn next very fine and dashed, corresponding to construction arcs and circles, and lastly the

Result straight line, to correspond to result arcs and circles.

The inking of a drawing is a recapitulation of each construction, and this important work should be executed with great care.

A postulate is a statement that something can be done, and is so evidently true as to require no reasoning to show that it can be done.

An axiom is a truth gained by experience, and requiring no logical demonstration.

A theorem is a truth requiring demonstration.

LINES AND ANGLES.

A right line is the shortest distance between two points.

When the term *line* alone is used, it indicates a *right line. A vertical line* is the "*plumb-line*"; *a horizontal line*, one making a right angle with the vertical and a line of any other direction, is called *oblique*.

A curved line or curve changes its direction in every point.

When two lines lying in one plane, on being produced in either direction, do not intersect, these lines are said to be *parallel.*

Two lines in one plane, which have a difference of direction, are said to form an *angle*. The point of intersection of these two lines, *called* SIDES, is the *vertex* of the angle.

When two such lines intersect each other, so that all four angles formed are equal, we say they are *right angles.* The common vertex of these four right angles may be assumed to be the center of a circle, which by diameters is divided into 360 equal parts, called *degrees* (°). Each angle coutains ¼ of 360° = 90°, which is the right angle. An angle greater than 90° is an *obtuse angle;* an angle smaller than 90° is an *acute angle.* When the two sides of an angle form a straight line, the angle is called a *straight angle*, and its magnitude is 180°.

Generally we designate an angle by three letters, for instance, *b a c* or *c a b;* then the middle letter (a) indicates the vertex, while the sides are *b a* and c *a*.

Alphabets for Titles of Technical Drawings.

Engineering Script, (Roman)

12345 abcdefghijklmnopqrstuvwxyz. 67890.

ABCDEFGHIJKLMNOPQRSTUVWXYZ.

ARCHITECTVRAL,

12345 abcdefghijklmnopqrstuvwxyz. 67890.

ABCDEFGHIJKLMNOPQRSTVVWXYZ.

Shop Skeleton.

abcdefghijklmnopqrstuvwxyz.

12345 ABCDEFGHIJKLMNOPQRSTUVWXYZ. 67890.

ABCDEFGHIJKLMNOPQRSTUVWXYZ

Round-Writing.

ABCDEFGHIJKLMNOPQRSTUVWXYZ.

abcdefghijklmnopqrstuvwxyz. 1234567890.

PLANES AND SURFACES.

A *plane* has two dimensions—length and breadth.

A *surface* is the boundary of a body.

Surfaces bounded by right lines are called *polygons*. *Regular polygons have equal sides and equal angles;* they are *equilateral* and *equiangular*.

POLYGONS ARE:

The triangle,	which has	3	sides,
" tetragon or quadrilateral,	"	4	"
" pentagon,	"	5	"
" hexagon,	"	6	"
" heptagon,	"	7	"
" octagon,	"	8	"
" enneagon or nonagon,	"	9	"
" decagon,	"	10	"
" undecagon,	"	11	"
" dodecagon,	"	12	" etc.

The triangles are: The *equilateral* triangle which is also *equiangular;* the *isosceles triangle*, having two sides equal, and the *scalene triangle*, whose sides are unequal.

An obtuse and a right-angled triangle have one obtuse and one right angle respectively. An acute angled triangle has three acute angles. The side or "leg" opposite the right angle in a right-angled triangle is called the hypotenuse, the sides or legs forming the right angle are the catheti.

The sum of the squares constructed on the catheti is equivalent to the square erected on the hypotenuse.

The sum of all angles in a triangle is equal to two right angles.

A line drawn from a vertex of a triangle perpendicular to the opposite or produced opposite side is called its *altitude or height.*

QUADRILATERALS.

The *square* has equal sides and 4 right angles.

The *rectangle* has opposite sides equal and 4 right angles.

The *rhombus* has equal sides and equal opposite angles.

The trapezoid has only two parallel sides.

The trapezium is an entirely irregular quadrilateral.

Quadrilaterals which have the opposite sides parallel are parallelograms.

CIRCLE.

Fig. I. Plate 15.—*Definition.*—A circle is a curve the points of which are equally distant from a fixed point, called the center.

The distance from the center to any point of the circle is called the *radius.* The connecting line of any two points of the circle is called a *chord.* If the chord is produced to any point outside the circle, it is called a *secant.* The chord through the center is called the *diameter.* The circle considered as a length, is called a *circumference.* Any arbitrary part of the circumference is called an *arc.* The arc that forms the fourth part of the circumference is called a *quadrant;* the sixth part a *sextant;* the eighth part an *octant;* while half the circumference is called a *semi-circle.* The area comprised by two radii and the intervening arc is called *sector;* the area comprised by a chord and the corresponding arc is called a *segment* of a circle.

In Fig. 1, C D, C B and C A are radii, G H is a chord, E I F is a secant, A B is a diameter, G J H an arc, area D C B L D is a sector, area H G J H a segment, tract A J D B a semi-circle.

Postulate.—Draw a circle, if the center and the radius are given

BLOCK,

ABCDEFGHIJKLMNOPQR
STUVWXYZ. 1234567890. &.

SHADOWLINE,

ABCDEFGHIJKLMNOPQR
STUVWXYZ. 1234567890. &.

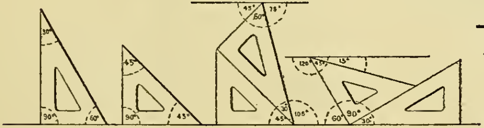


Fig 1. Fig 2. Fig 3. Fig 4

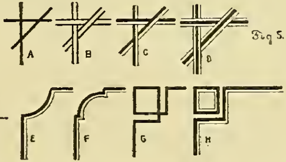


Fig 5.

CONSTRUCTIONS.

LINES.

1.—FIG. 1.—**Problem.**—*At a given point in a given line to erect a perpendicular, or to bisect a straight angle.*

Solution.—Let M N be the given line and A the given footpoint of a perpendicular. From A as a center and with any radius describe the circle B, C; B and C are equidistant from A and are the centers of arcs with equal radius greater than B A, which intersect at point D. Draw the line D A, which is perpendicular to the line M N, in point A.

2.—FIG. 2.—**Problem.**—*To draw from a given point a perpendicular to a given line.*

Solution.—With the given point A as a center describe a circle intersecting the given line M N in two points, B and C. From B and C as centers and with equal radii draw arcs intersecting at D. Connect points A and D by the line A D, which is perpendicular to M N.

3.—FIGS. 3, 4, 5 and 6.—**Problem.**—*To erect a perpendicular at the end of a given line,* M A.

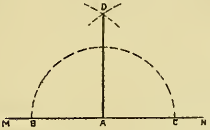
Solution.—Take any point C outside of M A as a center, and with a radius C A describe a circle intersecting M A at D. Draw the diameter D C B. Connect points B and A by the line B A, which is the perpendicular to M A.

4.—FIG. 4.—*Solution.*—From A as a center and any radius describe the circle B N, at which make B C = A B and pass through points B and C the line B C indefinite; make then C D = C B and connect A and D by the line A D, which is the required perpendicular.

5.—FIG. 5.—*Solution.*—Describe from A as a center and any radius the circle B C E. Make E C = C B = B A, and from E and C as centers and with equal radii draw intersecting arcs at D. Connect D with A with a line, and D A is the required perpendicular.

6.—FIG. 6.—*Solution.*—From A towards M lay down a division of 5 equal units. With A as a center and 3 units as a radius draw the arc 3 B indefinite, and 4 as center and 5 units as radius cut the arc at B. Connect B with A, and line B A is the required perpendicular.

Fig. 1.



At a given point in a given line to erect a perpendicular.

Fig. 2.

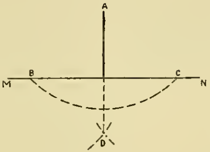


Fig. 3.

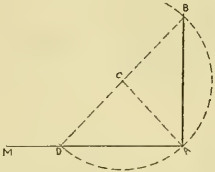


Fig. 4.

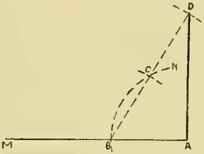


Fig. 5.

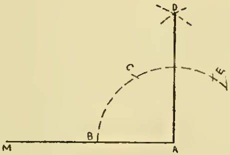
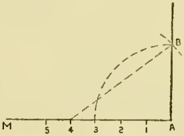


Fig. 6.



7.—FIG. 1.—**Problem.**—*To construct a perpendicular at or near to the end of a given line.*

Solution.—When M N is the given line, take in M N an arbitrary point A as a center and a radius longer than A N; describe arc C E D. From an other point, B, near N, with any radius, draw arcs intersecting circle (A) at C and D. Connecting points C and D by a line we have the required perpendicular.

DIVISION OF LINES.

8.—FIG. 2.—**Problem.**—*To bisect a line.*

Solution.—When A B is the given line, use A as center, and with a radius greater than ½ A B draw the arc D C E. With the same radius and center B draw an arc to intersect the arc D C E in points D and E, which are connected by the line D E. The line D E will not alone cut the line A B into two equal parts, but will also be a perpendicular to A B.

FIGS. 3, 4, 5 and 6.—**Problem.**—*To cut a given line into any number of equal or proportional parts.*

9.—FIG. 3.—**Problem.**—*A line* A B *shall be divided into* 7 *equal parts.*

Solution.—Draw the line B N at about 35° to A B. Lay thereon, starting from B, seven times a unit and connect points 7 and A by line 7 A. Parallel with line 7 A draw lines from each division point, 6, 5, 4, etc., which will divide A B into the required number of 7 equal parts. A C is $\frac{1}{7}$ of A B.

Remark.—Parallel lines are drawn with the set and T square combined. Adjust the longest side of the set square to coincide with the line with which we intend to draw parallels, and place the T to one of the right-angle sides of the set square. Keep T firmly in this position and slide the set square along its edge in the required direction and draw the parallels.

10.—FIG. 4.—**Problem.**—*To cut a given line into two proportional parts, as* 3:8.

Solution.—Draw the line B N, and from B lay down a division of 3+8 equal parts. Connect points A and 11 by the line A 11 and draw parallel with it 3 C. C B is $\frac{3}{11}$ and C A $\frac{8}{11}$ of A B.

11.—FIG. 5.—**Problem.**—*To cut a given line into three proportional parts, as* 7:3½:1¾.

Solution.—Draw the line B N, and from B lay down a division of 7+3½+1¾ equal parts. Connect point 12¼ with A and draw parallel with 12¼ A, the lines 10½ C and 7 D. A C is then 1¾, C D 3½, and D B 7 parts of line A B.

12.—FIG. 6.—**Problem.**—*To cut a given line into any number of equal parts by a scale.*

Solution.—Draw a rectangle A 0 14 N, and divide A N by horizontals into any number of equal parts, and number them 0, 1, 2, 3, 4, 5, etc. When, f. i., the line A B is to be divided into 9 equal parts, take the line to be divided as a radius and A as center; describe an arc to intersect line 9 at point B 9, which connect with A by line B 9 A. By the horizontals the line B 9 A is divided into 9 equal parts. In the same figure the problem is solved to divide the line A B into 11½ equal parts.

Plate 5.

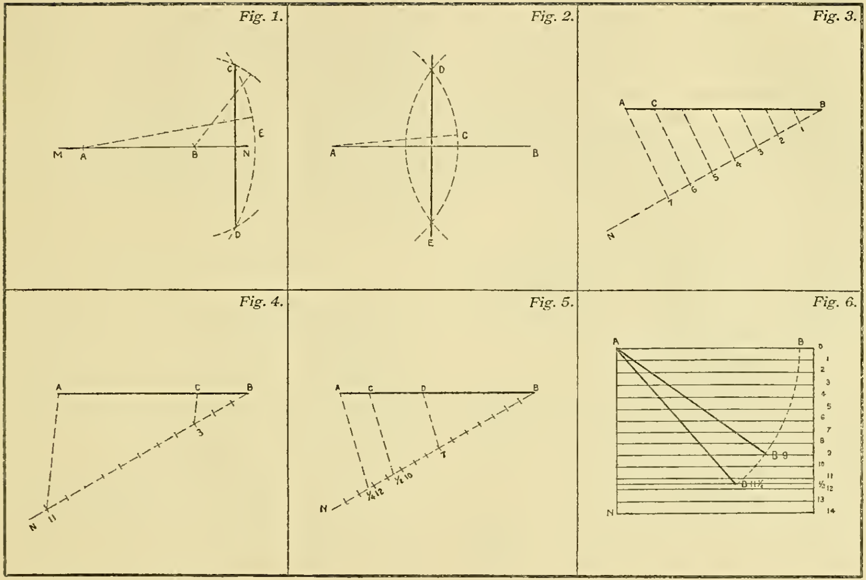


Fig. 1.
C
E
M A B N
D
Fig. 2.
D
C
A B
E
Fig. 3.
A C B
1 2 3 4 5 6 7
N
Fig. 4.
A C B
3
N 11
Fig. 5.
A C D B
7
½ 10
¼ 12
N
Fig. 6.
A B
0 1 2 3 4 5 6 7 8 9 10 11 ½ 12 13 14
B 9
D 11¼
N

SOLUTION OF ANGLES.

13.—Fig. 1.—**Problem.**—*To construct an angle equal to a given one.*

Solution.—Angle C A B is the given angle. When the vertex O and one side O N of the angle to be constructed are given, describe with O as a center and A C as a radius the arc E D, and from D as a center with the radius B C the arc at E; draw the line E O. Angle C A B = angle E O D.

14.—Fig. 2.—**Problem.**—*To bisect an angle.*

Solution.—Let B A C be the given angle. With A as a center and a radius A B draw the arc B C. B and C are the centers for arcs with equal radii, intersecting at D; draw line D A, which divides B A C into two equal parts.

15.—Fig. 3.—**Problem.**—*To trisect a right angle.*

Solution.—From vertex A, with the radius A B, draw the arc B C. With B as center and the same radius draw the arc A E, and from C the arc A D; draw lines D A and E A. Angle B A D = D A E = E A C.

16.—Fig. 4.—**Problem.**—*To trisect any angle.*

Solution by Dr. Henry Eggers.—Let C A B be the angle to be trisected. Describe with A as center a semicircle B C D, which intersects the prolonged side B A of tbe angle at D; draw from C an arbitrary line C E M and make E F = E A, and draw F G C; then make G H = G A and draw H I C. An additional operation will not be necessary, as the lines will fall so close together as to almost coincide, and it is angle C H B which equals ⅓ C A B. This construction is convenient for angles up to 90°; and in case of the trisection of an obtuse angle we bisect first and then trisect, so that the double third of the bisected angle is equal to the third of the given obtuse angle.

SOLUTION OF TRIANGLES.

17.—Fig. 5.—**Problem.**—*To construct a triangle when the three sides are given.*

Solution.—Lines 1, 2 and 3 are the sides given. Lay down line B C = line 1. From C as center, with line 2 as a radius, draw an arc, and with line 3 as radius and center B another arc, intersecting the first arc at D. Draw lines D C and D B; then D C B is the required triangle.

18.—Fig. 6.—**Problem.**—*To construct a triangle of which two sides and the included angle are given.*

Solution.—Construct angle D, and from its vertex cut off the sides 1 and 2, that is C B and C E, and draw line E B; then E B C is the required triangle.

Fig. 1.

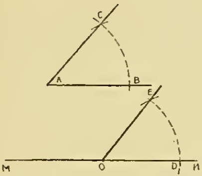


Fig. 2.

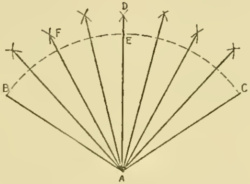


Fig. 3.

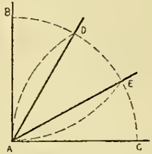


Fig. 4.

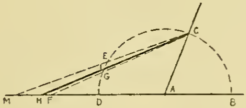


Fig. 5.

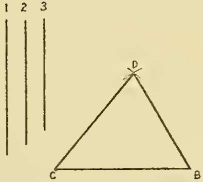
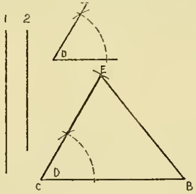


Fig. 6.



19.—FIG. 1.—**Problem.**—*To construct a triangle of which one side* (1) *and the two adjacent angles* D *and* E *are given.*

Solution.—Lay off C B equal to line 1; transfer the angles D and E on line C B, and prolong the sides to intersect at F; then triangle C F B is the required triangle.

20.—FIG. 2.—**Problem.**—*To construct a triangle of which one side* (1), *one adjacent angle* D *and one opposite angle* E *are given.*

Solution.—Construct C B equal line (1) and angle D at C as before; at an arbitrary point E on line C M draw angle C M N = E, and parallel with M N the line B F. F is the third vertex of the required triangle C F B.

PROPORTIONAL LINES.

21.—FIG. 3.—**Problem.**—*To construct to three given lines a fourth proportional.*

Solution.—Lay down an angle M A N of about 40°, and from A cut the segments A 1 = line 1, A 2 = line 2, A 3 = line 3; draw line 2 1, and with it parallel the line 3 x. A x is the required line. 1 : 2 = 3 : A x.

22.—FIG. 4.—**Problem.**—*To construct to two given lines a third proportional.*

Solution.—Lay down the angle as before, and from A cut the segments A 1 = line 1, A 2 = line 2, A 2′ = line 2. Draw line 2 1, and parallel with it 2′ x. A x is the required line. 1: 2 = 2:A x.

23.—FIG. 5.—**Problem.**—*To construct a mean proportional to two given lines.*

Solution.—A B + B C is the sum of the given lines 1 + 2. Find point D, the center of A C, and a radius D A; draw the semi-circle A X C. Erect at B a perpendicular B X, which is the required line. A B : B X = B X: B C.

24.—FIG. 6.—**Problem.**—*To construct to a given line major and minor extreme proportionals.*

Solution.—At point B of the given line A B erect a perpendicular B D = ½ A B, and draw line A D F indefinite; with D as center, D B as radius, describe semicircle E B F, and from A as center, A E as a radius, draw arc E X. The line A F : A B = A B : A X.

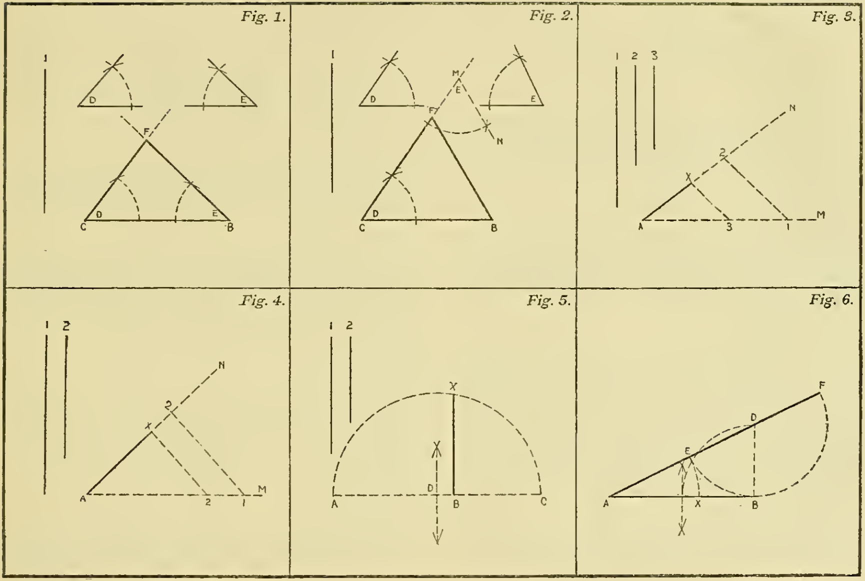


Fig. 1.
1
D
E
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D
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C
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Fig. 2.
1
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Fig. 3.
1 2 3
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2
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A
3
1
M
Fig. 4.
1 2
N
2
X
A
2
1
M
Fig. 5.
1 2
X
D
A
B
C
Fig. 6.
F
D
E
A
X
B

25.—Fig. 1.—**Problem.**—*To construct a regular triangle on a given base.*

Solution.—A B is the given base. With A and B as centers and A B as radius draw arcs intersecting at C. Draw the lines C A and C B. A C B is the required regular triangle.

26.—Fig. 1.—**Problem.**—*To construct a regular hexagon on a given base.*

Solution.—Let A B be the given base. Construct on this a regular (or equilateral) triangle. The vertex C is the center, and C A = C B the radius of a circle, in which a regular hexagon A B D E F G, with A B as side, can be inscribed.

Corollary.—A regular hexagon may be divided into six equal equilateral triangles, the common vertices of which lie in the center of it.

27.—Fig. 2.—**Problem.**—*To construct a regular heptagon at a given base.*

Solution.—Draw with the given base A B the equilateral triangle A 6 B, as in the previous construction. From center D of A B draw the line D 6 12 perpendicular to A B. Divide 6 A into six equal parts. These parts transfer on line 6–12 and number them, 7, 8, 9, 10, 11 and 12. Point 7 is the center, and 7 A the radius of a circle, in which the regular heptagon A B C D E F G, with A B as side, can be inscribed.

28.—Fig. 2.—**Problem.**—*To construct a regular polygon with more than 6 sides.*

Solution.—With points 7, 8, 9, 10, 11 and 12 as centers, and 7 A, 8 A, 9 A, 10 A, 11 A and 12 A, respectively as radii, draw circles in which the line A B as repeated chord will form the regular heptagon, octagon, enneagon, decagon, undecagon and dodecagon.

Remark.—Regular polygons with greater number of sides are rarely used in practice, and are therefore omitted here.

29.—Fig. 3.—**Problem.**—*To construct a square at a given base.*

Solution.—Let A B be the given base. Draw at A and B perpendiculars with set and T square, and make A C = A B, and with T square draw C D. A C D B is the required square.

30.—Fig. 3.—**Problem.**—*To construct a regular octagon at a given base.*

Solution.—In the bisecting point H of the given base A B erect a perpendicular, H F, at which make H E = A H and E F = E A. F is the center and F A the radius of a circle, in which draw A B eight times, as repeated chord, to complete the required octagon A B G H I J K L.

31.—Fig. 4.—**Problem.**—*To construct a regular pentagon at a given base.*

Solution.—Let A B be the given base; produce it towards N. Erect at B a perpendicular, B D = A B. Bisect A B by point C; with C as center and C D as radius draw arc D E. With A and B as centers and A E as radius draw arcs to intersect at F. With F and A as centers draw arcs intersecting at G; and from F and B as centers, with the same radius A B, draw arcs intersecting at H. Connecting B H, H F, F G and G A by lines we complete the required pentagon A B H F G.

32.—Fig. 4.—**Problem.**—*To construct a regular decagon at a given base.*

Solution.—Let A B be the given base. Follow the construction of the pentagon until the position of point F is found; this is the center, and F A the radius of the circle, in which as repeated chord the line A B will complete the required regular decagon A B I J K L M N O P.

33.—Fig. 5.—**Problem.**—*To construct triangles equivalent to a given one.*

Solution.—Let A C B be the given triangle; draw line M N parallel with A B through point C. Locate an arbitrary point E or G in line M N, and draw lines E A and E B, and G A and G B. Triangle A E B = A C B = A G B. If one side of the triangle is called the base, a perpendicular drawn from the opposite vertex to the base, or produced base, is the altitude or height of the triangle, as E F, C D and G H.

Theorem.—Triangles of equal base and altitude are equivalent.

34.—Fig. 5, A.—**Problem.**—*To construct parallelograms equivalent to a given one.*

Solution.—Let A B D C be the given parallelogram, with base A B. Draw the line M N parallel with A B, make E F and G H = C D, and draw lines E A, F B, G A and H B. The parallelogram E F B A = C D B A = G H B A.

In a polygon any right line which passes through two nonconsecutive vertices of its circumferential angles is called a *diagonal.*

Theorem.—Either diagonal divides the parallelogram into two equal triangles.

35.—Fig. 6.—**Problem.**—*To construct a rectangle equivalent to a given triangle.*

Solution.—A B C may be the given triangle, and C F its altitude. Bisect C F rightangularly by line D E, and erect the perpendiculars B E and A D. A D E B is the required rectangle.

Plate 8.

Fig. 1.

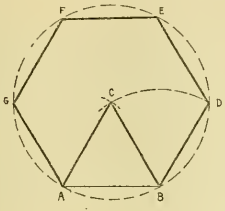


Fig. 2.

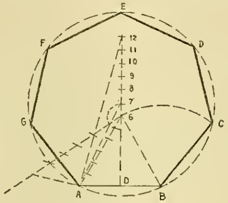


Fig. 3.

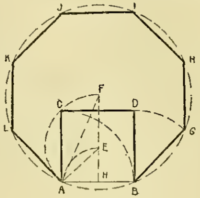


Fig. 4.

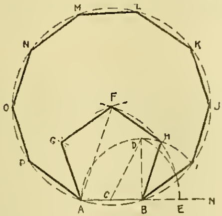


Fig. 5.

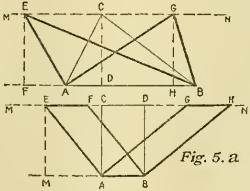
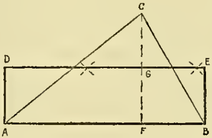


Fig. 5. a.

Fig. 6.



Hanstein's Constructive Drawing.

36.—FIG. 1.—**Problem**.—*To construct a rectangle equivalent to a given trapezoid.*

Solution.—Let A B C D be the given trapezoid. Bisect rightangularly its altitude L M by the line I K, which bisects also the sides B A and C D in I and K. Perpendicular to I K, through I and K, draw F G and E H. F E H G is the required rectangle, equivalent to the trapezoid A B C D.

37.—FIG. 2.—**Problem**.—*The side of a square is given: to construct the sides of squares that are twice, three times, four times, etc., as great as the square over the given line.*

Solution.—Construct a right angle B A 1; make B A and A 1 equal to the given side of the square; then lay off successively A 2 = B 1, A 3 = B 2, A 4 = B 3, etc. A 2, A 3, A 4, etc., are the sides of squares that are respectively twice, three times, four times, etc., the area of the square over A 1.

38.—FIG. 3.—**Problem**.—*To construct a triangle equivalent to a given irregular pentagon.*

Solution.—Let A B C D E be the irregular pentagon. By the diagnols A C and C E divide it into three triangles A B C, C A E and C D E. Produce the base A E to the left and right indefinitely, and parallel to C A draw the line B F; connect C with F; then draw D G parallel with C E and connect C with G. The sum of the triangles C F A + C A E + C E G is equal to the triangle C F G, which is equivalent to the irregular pentagon A B C D E.

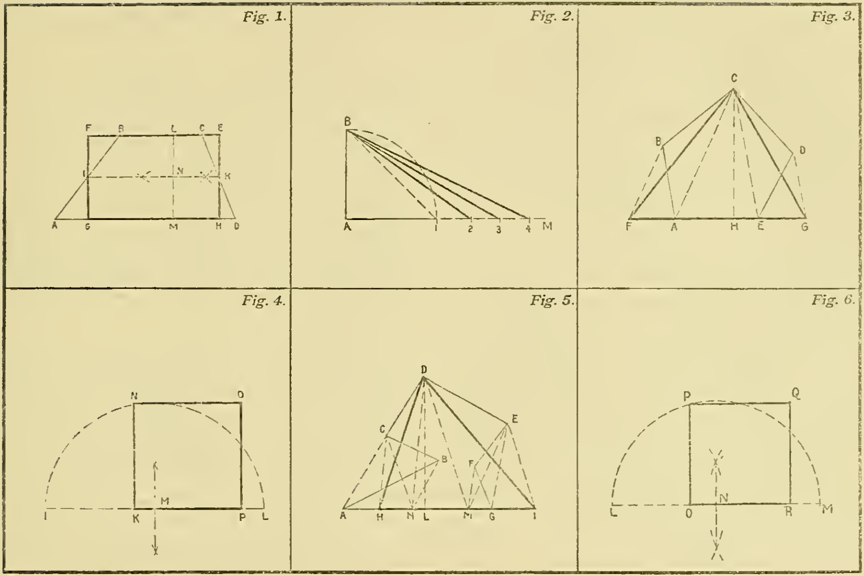
39.—FIG. 4.—**Problem**.—*To construct a square equivalent to a given triangle.*

Solution.—Let C F G, Fig. 3, be the given triangle. Construct a mean proportional between half the base F G and altitude C H, as shown in Fig. 5, Plate 7, by making I K = $\frac{1}{2}$ F G, and K L = C H. The sum I K + K L is the diameter of the semicircle I N L. Erect at K a perpendicular, which is intersected by the circle in N. N K is the required side of the square, and N O P K is the square, which is equivalent to the triangle C F G and the irregular pentagon A B C D E.

40.—FIGS. 5 and 6.—**Problem**.—*To transform an irregular heptagon into an equivalent triangle and square.*

Solution.—Let A B C D E F G be the irregular heptagon. Draw line C A, and parallel to it B N; connect N and C by line N C. Triangle C N A = C B A. Treat the triangle E F G in a similar way, and you have transformed the heptagon into the irregular pentagon N C D E M. Proceed as in Fig. 10, and transform the pentagon into the triangle D H I; transform this into the square P Q R O, which then is equivalent to the given heptagon A B C D E F G.

Plate 9.



Hanstein's Constructive Drawing.

41.—FIGS. 1 and 2.—**Problem.**—*To construct a polygon equal to a given one by triangles.*

Solution.—Polygon Fig. 1 is given and divided into triangles, A B C, etc., etc. Draw line A′ B′ parallel and equal to A B. On A′ B′ construct the triangle A′ B′ C′ equal to triangle A B C. The remaining triangles of Fig. 1 lay off in the same order and position as Fig. 2 shows, starting from side B C_1; then polygon (Fig. 2) is the required one.

42.—FIGS. 3 and 4.—**Problem.**—*To construct a polygon equal to a given one by sectors.*

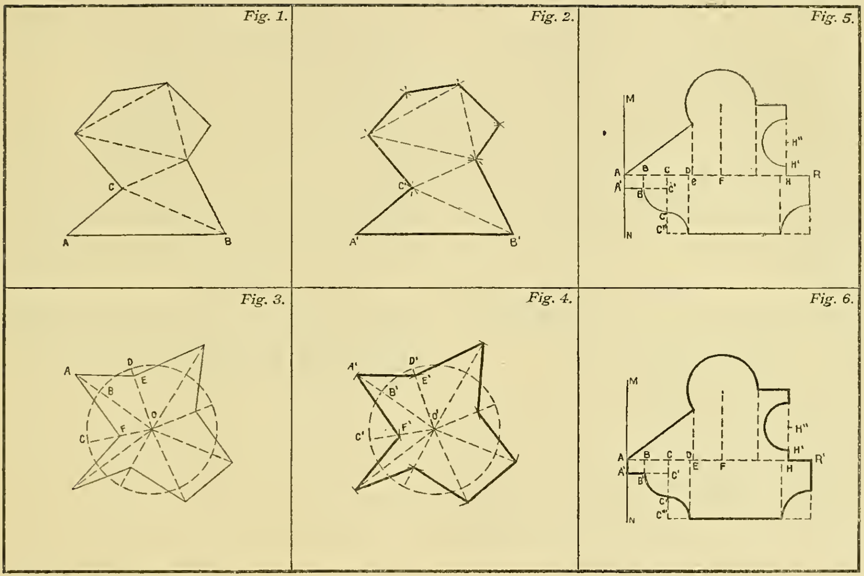
Solution.—Polygon Fig. 3 is given. From center O with any radius describe circle C B D, etc., and draw from center O a radius to each vertex of the polygon to intersect with the circle. Locate center O′, Fig. 4, and with radius O′ D′ equal O D describe the circle D′ B′ C′, etc., and draw O′ D′ parallel to O D; make arcs D′ B′ = D B, B′ C′ = B C, etc., and pass lines through points D′, A′, C′, etc.; further make O′ E′ = O E, O′ A′ = O A, O′ F′ = O F, etc., and by connecting points E′ A′ F′, etc., complete the required polygon.

43.—FIGS. 5 and 6.—**Problem.**—*To construct a polygon equal to a given one, by co-ordinates.*

Remark.—In the plane of drawing a convenient line is drawn (horizontal), called the *axis of abscissae;* the position of the different vertices of the given figure is determined by perpendiculars, called *ordinates*, from these vertices to the axis of abscissae. Take any convenient point, A, on this axis and draw a perpendicular to it, M N. This line is called the axis of ordinates, and reckoned from this point A (called the origin) the segments determined by the foot-points of the ordinates are called *abscissae*. The common appellation of both systems of lines (the abscissae and ordinates) is *co-ordinates.*

Solution.—Fig. 5 is the given polygon. Through any vertex (origin) draw a horizontal, A R, and perpendicular to it the ordinates from each vertex or principal point for transmission. Draw A R′, Fig. 6, and lay off A B, A C, A D, etc., = A B, A C, A D, etc., of Fig. 5. Erect the perpendiculars A A′, B B′, C C′ C″ C‴, etc., and make A A′, B B′, C C′, C C″, C C‴, etc., equal to the corresponding perpendiculars in Fig. 5. Connect A and A′, A′ and B′, describe with radius C′ C, center C′, arc B′ C″, etc., and complete the required polygon, Fig. 6.

Plate 10.



Hanstein's Constructive Drawing.

44.—FIGS. 1 and 2.—**Problem.**—*To construct a polygon equal to a given one, radiating in a circle.*

Solution.—Let A E D G, etc., be the given polygon. Describe with A D, A E, etc., as radii and A as center the circles C D, E F G, etc., and make F′ E′ = F E, F′ G′ = F G, etc. Connect D′ and E′, D′ and G′, etc., and complete the required polygon. Fig. 2 shows the construction applied to other polygons.

Remark.—This construction is used conveniently to draw a *rosette* in which an ornamental unit occupies a sector division of a circle.

45.—FIGS. 3 and 4.—**Problem.**—*To construct symmetric polygons or outlines.*

Solution.—Let L M, etc., be the given outline as a profile of the base of a column. Draw the horizontals L L′, M M′, etc., and the axis of symmetry R N. Make A L′ = A L, B M′ = B M, D O′ = D O, etc. Connect L′ and M′, etc., and complete the required symmetric profile of the base of the column.

46.—FIGS. 5 and 6.—**Problem.**—*To construct an irregular outline equal to a given one.*

Solution.—Let B A C be the given outline. Cover this with a series of equal small squares and construct in Fig. 6 the same number of equal squares arranged as in Fig. 11, and transfer the points of intersections of the irregular outline with the sides of the squares; make M′ A′ = M A of Fig 5, and M′ B′ = M B, etc. Connect B′ A′ C′ by a free-hand line and complete the required irregular outline, Fig. 6.

Fig. 1. *Fig. 2.*

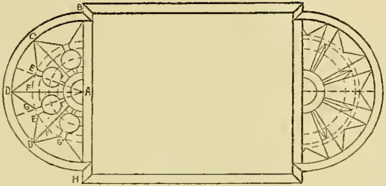


Fig. 5.

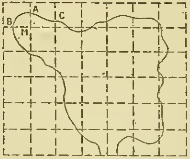


Fig. 3. *Fig. 4.*

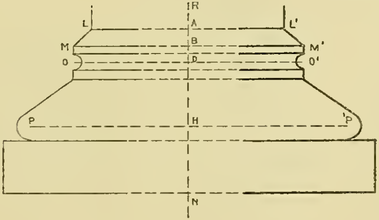
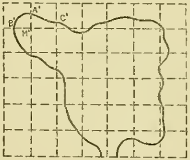


Fig. 6.



47.—FIGS. 1, 2 and 3.—**Problem.**—*To construct a polygon similar to a given one of $\frac{4}{7}$ its circumference.*

Solution.—Let D A B C, etc., Fig. 1, be the given polygon. Construct the Scale Fig. 2. A perpendicular O 7, longer than the longest side of the given polygon, is divided into 7 equal parts; draw a horizontal line O N of an arbitrary length and connect points 7 and 4 with N by the lines 7 N and 4 N. O 4 is $\frac{4}{7}$, — 4 7 is $\frac{3}{7}$ of the line O 7. All lines between O N and 7 N and parallel to O 7 are divided by 4 N and 7 N in the same proportion. To obtain the length of A' B', Fig. 3, place line A B in the scale as indicated by line A B' B, of which A B' is $\frac{4}{7}$ of line A B. Transfer the remaining sides of the polygon by parallels and find of each the proportionate length in the scale Fig. 2, as shown by line A B; D' A' B' C', etc., is the required polygon.

48.—FIGS. 1, 4 and 5.—**Problem.**—*To construct a polygon similar to a given one, having $\frac{4}{7}$ its area.*

Solution.—Let D A B C, etc., Fig. 1, be the given polygon. On a horizontal line O 4 lay down a division of 7+4 equal parts and make O 4 the diameter of a semi-circle O M 4. Erect at point 7 the perpendicular 7 M and draw lines M O and M 4. Then make line M B' equal to A B of the given polygon and draw B' B'' parallel to O 4; A'' B'' (Fig. 5) = M B'' in the scale Fig. 4. In relation to the side A B of the given polygon, A'' B'' is the side of a polygon, whose area is $\frac{4}{1}$ of the given one. Treat the remaining sides of the polygon similar to the side A B and complete the required polygon D'' A'' B'' C'', etc.

49.—FIG. 6 —**Problem.**—*To construct similar polygons which have $\frac{3}{5}$ the circumference and $\frac{3}{5}$ the area of a given one.*

Solution for circumference reduction.—Let D C B A E, etc., be the given polygon. From any point O therein draw radii to the vertices D C B A E, etc., and divide any one radius (O D) into 5 equal parts. Parallel to D C from point 3 draw D' C', with O B, C' B', etc., and D' C' B' A E, etc., is the required polygon.

Solution for area reduction.—Make radius O D the diameter of the semi-circle O N D and erect at division point 3 the perpendicular 3 N and draw N O. Make O D'' = O N and proceed as before in drawing D'' C'' parallel with D C, C'' B'' with C B, etc. D'' C'' B'' A'' E'', etc., is the required polygon.

50.—FIGS. 7, 8 and 9.—**Problem.**—*To reduce any irregular outline in proportion 8:5.*

Solution.—Let G H I K be the given irregular outline. Cover the given outline by a net of equal squares, the sides of which we reduce by the scale, Fig. 8, to A' B' = ⅝ of A B. Draw with A' B' as unit the same number of squares as in Fig. 7. Transfer the points of intersection of the irregular outline with the sides of the squares, in reducing their distances from the vertices of the squares by scale Fig. 8, and transfer into Fig. 9. Connect these points by a free-hand line, which is the required reduction of the irregular outline.

Treat the surface reduction, Fig. 11, with the assistance of the scale Fig. 10 in a similar way, and we obtain the reduction in area.

Fig. 1.

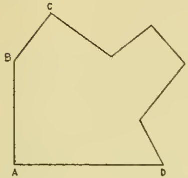


Fig. 2.

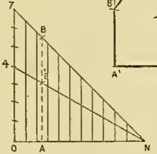


Fig. 3.

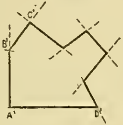


Fig. 4.

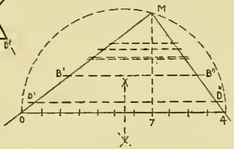


Fig. 5.

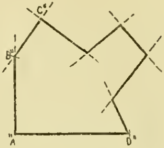


Fig. 6.

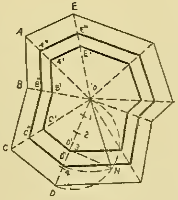


Fig. 7.

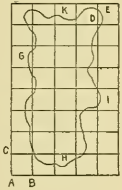


Fig. 10.

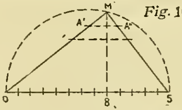


Fig. 8.

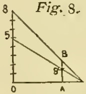
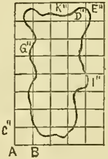


Fig. 9

Fig. 11.



51.—FIGS. 1, 2 and 3.—**Problem.**—*To construct a polygon similar to a given one, and of $\frac{3}{2}$ its circumference.* (*Transfer by triangles.*)

Solution.—Let A B D C, etc., be the given polygon. Construct the linear scale in proportion 2:3 Fig. 2 similar to Fig. 2, Plate 12, and divide the given polygon by diagonals into triangles. Line A B′ in the scale (Fig. 2) = A′ B′ of the polygon Fig. 3, whose circumference contains 3 units to 2 of the given polygon. Transfer and complete by triangles the required polygon A′ B′ D′ C′, etc., Fig. 3.

52.—FIGS. 1, 4 and 5.—**Problem.**—*To construct a polygon similar to a given one, which contains 3 to each 2 square units of the given polygon.*

Solution.—In the scale Fig. 4 the diameter of the semicircle consists of 2 + 3 equal parts; erect 2 M. Draw M O and M 3. Make M B′ Fig. 4 = A B of the given polygon and draw B′ B″ parallel to O 3, A″ B″ = M B″ of the polygon, Fig. 5, whose area has 3 square units to 2 of the given polygon.

Transfer and complete by triangles the required polygon A″ B″ D″ C″, etc., Fig. 5.

SCALES.

53.—FIG. 6.—**Problem.**—*To construct a scale of decimal division.*

Remark.—Small subdivisions of a unit which we cannot accurately perform with the dividers are constructed in Figs. 1 and 2.

Solution.—Let line A, 7 = 8 centimeters, AO=AB = 1 cm. The decimal subdivision (millimeter, mm) is obtained by dividing O N into 5 equal parts by the horizontals in points A, B, C and D. Bisect B N and draw lines 5 A and 5 O; line A 1 = $\frac{1}{10}$, B 2 = $\frac{2}{10}$, C 3 = $\frac{3}{10}$, etc., of O A, or 1, 2, 3, etc., mm. the required division.

54.—FIG. 6 A.—**Problem.**—*To divide a centimeter into 100 equal parts.*

Solution.—Upon a straight line A 7 lay off eight units (cm) and construct squares on these distances. Let first square be A B N O. Divide sides A B and B N into ten equal parts (mm). Draw horizontals through division points on A B. R, being the first point of division from N to B, is connected with O, and through the other points parallels to R O are drawn between B N and A O. These parallels subdivide the millimeter (mm) into tenths.

Fig. 1.

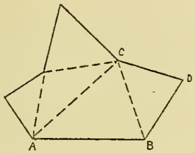


Fig. 2.

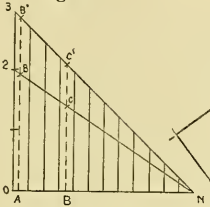


Fig. 3.

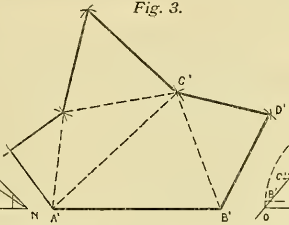


Fig. 4.

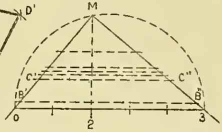


Fig. 5.

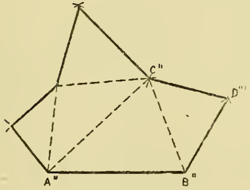


Fig. 6.

Meter.

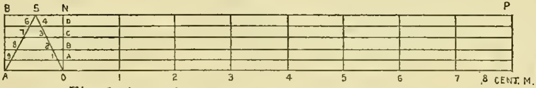
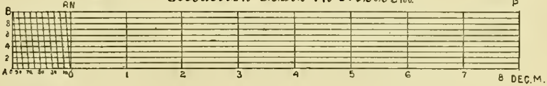


Fig. 6. A Reduction Scale in Meter $\frac{1}{100}$.



55.—FIG. 1.—**Problem.**—*To construct a scale in which an inch is divided into 64ths.*

Solution.—Let A 2 = 3 inches. Divide B N and B A into 8 equal parts each and complete the scale in the manner explained in Problem 54, Plate 13, Fig. 6 A. Line R O divides line R N = $\frac{1}{8}$ in. into 8 equal parts, hence into 64ths. Example: Take from this scale a line of $1\frac{37}{64}$ inch ($\frac{37}{64} = \frac{4}{8} + \frac{5}{64}$). From O to 4 = $\frac{4}{8}$ in.; follow the oblique line upward to the 5th horizontal point, N. Line N A = $\frac{4}{8}$ in., A B = $\frac{5}{64}$ in. and B M = 1 inch and line N M = $1\frac{37}{64}$ in., as required.

REDUCTION SCALES.

56.—FIGS. 2 and 3.—**Problem.**—*To construct a decimal reduction scale and draw by co-ordinates a polygon whose equations are indicated at tables* A *and* B, *Fig.* 2 A.

Remark.—To draw the scale and polygon in convenient proportion let the unit O A = $2\frac{1}{4}$ in., which may represent 100 feet.

Solution.—Let O A be the unit to represent 100 ft. in the decimal reduction scale and let A 200 = 3 such units. Divide O A, A B and B N into 10 equal parts, draw horizontals from 9, 8, 7, etc., and the oblique parallels with R O from division points 10, 20, 30, etc., and we have the required decimal scale. Example: Take from this scale a line to represent 173 feet. Begin at point O, pass to the left to 70, then upward the oblique line to the third horizontal point R. Line R A = 70 ft. A B = 3 ft. and B S = 100 ft., and R A + A B + B S = 173 feet.

The polygon, Fig. 3, is constructed with this scale.

Remark.—If the scale, Fig. 2, is used as a reduction scale in which O A represents 1 ft., we shall have to divide O A into 12 equal parts (inches), etc., and the scale will represent $\frac{1}{12}$ of actual dimension.

Fig. 1. Inch divided in 64th

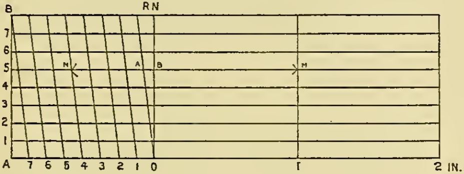


Fig. 2. A

Table A

AB	=	5. Ft
AC	=	27 "
AD	=	48.5 "
AE	=	58. "
AF	=	105.5 "
AG	=	109. "
AH	=	117. "
AI	=	128. "
AJ	=	139. "
AK	=	152 "
AL	=	163. "
AM	=	221. "

Table B

BB'	=	36 5. Ft
CC'	=	51. "
DD'	=	7 5 "
EE'	=	25 5 "
FF'	=	32 "
GG'	=	63. "
HH'	=	50.5 "
II'	=	32 "
JJ'	=	50.5 "
KK'	=	31.5 "
LL'	=	109.5 "
MM'	=	9 "

Fig. 2. Reduction Scale in feet, $\frac{1}{1200}$

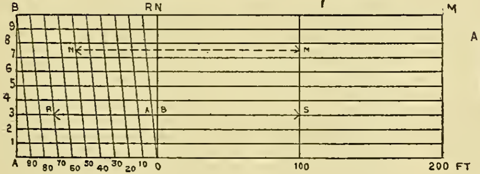
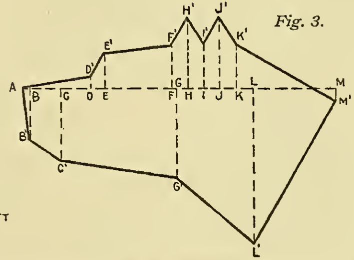


Fig. 3.



57.—FIG. 2.—**Problem.**—*To inscribe a regular triangle, hexagon and dodecagon in a given circle.*

Solution.—Let A B F D be the given circle. Describe with point A as a center and radius A C the arc B C D and draw line B D, which is the side of the required regular inscribed triangle.

Hexagon.—Line B A = radius, B C = the side of the required regular inscribed hexagon.

Dodecagon.—Bisect the arc B A by point E; draw B E, which is the side of the required regular inscribed dodecagon.

58.—FIG. 3.—**Problem.**—*To inscribe in a given circle, C, a square, octagon and a regular polygon of 16 sides.*

Solution.—Construct two perpendicular diameters, A B and D G. Draw D B, which is the side of the inscribed square.

Octagon.—Bisect the quadrant D A (in E) and draw D E, which is the side of the required regular inscribed octagon.

The regular polygon of 16 sides.—Bisect the arc D E by F, and draw D F, which is the side of the required regular inscribed polygon of 16 sides.

59.—FIG. 4.—**Problem.**—*To inscribe a regular pentagon and decagon in a given circle.*

Solution.—Draw two perpendicular diameters, A B and E I, in the given circle C. Bisect radius C B at point D, and with D E as radius, D as center, describe arc E F and draw line E G = E F, which is the side of the required regular inscribed pentagon.

Decagon.—Bisect the arc E G by point H and draw E H, which is the side of the required regular inscribed decagon.

60.—FIG. 5.—**Problem.**—*To inscribe a regular heptagon and a regular polygon of 14 sides in a given circle.*

Solution.—Draw a radius, A C. With point A as center and A C as radius describe arc B C D and draw B E D. H F = F D = D E = the side of the regular heptagon in the given circle.

Bisect arc F H by point G and draw H G, which is the side of the regular polygon of 14 sides in the circle.

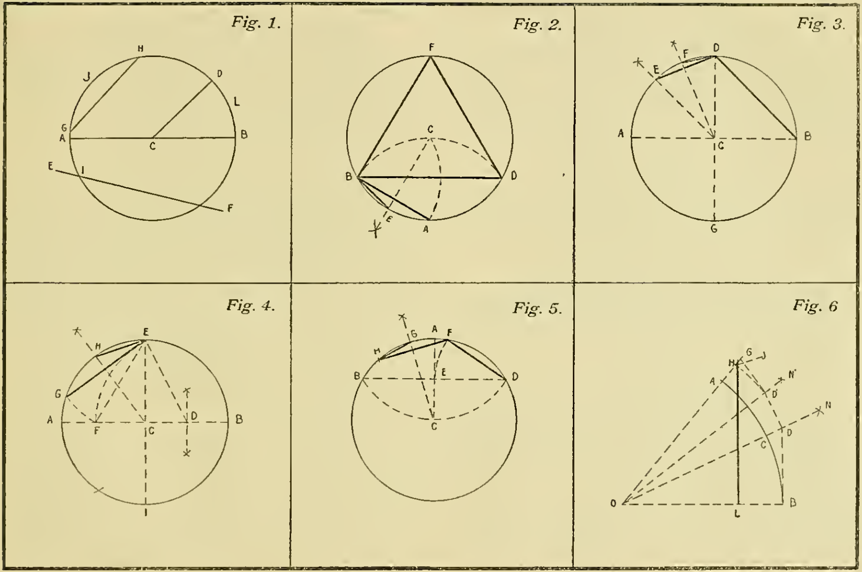
RECTIFICATION OF ARCS.

61.—FIG 6.—**Problem.**—*To rectify a given arc.*

Solution.—Let A B, corresponding to angle A O B, be the given arc. Bisect angle A O B by O N and bisect also angle A O N by O N′. Erect B D perpendicular to O B at B, D′ D perpendicular to O N at D, D′ G perpendicular to O N′ at D′, and draw arc D′ H with radius O D′ and center O. Divide H G into three equal parts, and from the first division point J, near H, drop J L, a perpendicular to O B, then J L = arc B C A. The approximation is very close as long as the given angle does not exceed 60°; but for greater angles, the half of them may be rectified.

From the rectified arc we can find the area of the corresponding sector: construct a triangle with the rectified arc J L as base and with the radius of the circle as the altitude; this triangle has the same area as the sector in question.—To transform a circle into an equivalent square, we may rectify the arc of 45°, construct a triangle that has for a base 8 times the length of this arc, and for the altitude the radius. Transform this triangle into a square, then this square will be equal to the area of the circle.—In order to find the length of the circumference of a circle we would rectify the arc of 45° and multiply this length by 8

Plate 15.



Hanstein's Constructive Drawing.

62.—Fig. 1.—**Problem.**—*To construct a line equal to the semi-circumference of a given circle.*

Solution.—In the given circle C draw two perpendicular diameters, A B and F G, and, at G, the indefinite line E H perpendicular to F G. With A as center and A C as radius describe arc C D and draw line C D E. Make E 3 = 3 A C and draw F 3 = G H, which is equal to the semi-circumference of the circle C. Calculation gives—

F 3 = 3.14153 times radius;
error = 0.00006 of semi-circumference.

Denoting the ratio of the circumference to the diameter of a circle by the letter π, then this ratio has been more accurately found to be

$$\pi = 3.1415926;$$

for common usage it suffices to take for it—

$$\pi = \tfrac{22}{7} = 3.1428, \text{ with an error } = 0.001.$$

Among the many approximative methods to rectify a circle, the above method has the advantage that it can be performed with one opening of the compasses.

TANGENTS.

63.—Fig. 2.—**Problem.**—*To construct a tangent at a given point of a circle.*

Definition.—A tangent is a line touching the circumference of a circle in one point only, the point of contact, and is a perpendicular to a radius, drawn to the point of contact.

Solution.—Let C be the given circle and A the point of contact. Draw the radius C A, and perpendicular to it, at point A, the line M N, which is the required tangent.

64.—Fig. 3.—**Problem.**—*From a given point outside a circle to draw tangents to this circle.*

Solution.—Let C be the given circle and A the outside point. Draw A C, and on A C as a diameter describe a circle, center B; this circle B intersects circle C at points O and P; then lines A O and A P are tangents to circle B.

65.—Fig. 4.—**Problem.**—*To construct common exterior tangents to two given circles.*

Solution.—Let C and A be the given circles. Draw line C A and upon this as diameter, circle C A; with the difference C F, of the radii D A and C E draw H F I from center C, intersecting circle B at points H and I. Draw radius C O through H, and C P through I. Radii A O′ and A P′ are parallel to C O and C P respectively. O′ O and P′ P are the points of contact of the common tangents.

66.—Fig. 5.—**Problem.**—*To construct common interior tangents to two circles.*

Solution.—Follow the previous construction and describe the circle C F A G. With the sum of the radii of both circles A D + C I = C E draw arc F E G; also the lines C F and its parallel radius A P′, and C G and its parallel A O′. The intersections O and O′, P and P′ are the points of contact of the required tangents P P′ and O O′.

Plate 16.

Fig. 1.

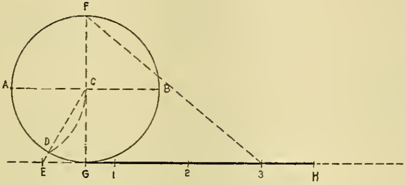


Fig. 2.

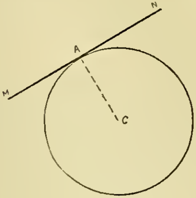


Fig. 3.

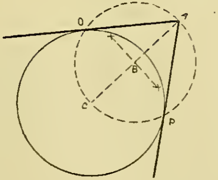


Fig. 4.

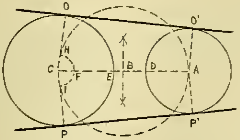
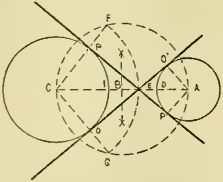


Fig. 5.



67.—FIG. 1.—**Problem.**—*To construct circles,* D *and* H, *that touch a given line,* M N, *and a given circle in point* A.

Solution.—Draw line H C A B through center C and the given point of contact A; at A erect a perpendicular to H B, intersecting M N in point E. With E as center, E A as radius, draw the semicircle F A G and erect at F and G perpendiculars to M N, to obtain on line H B the intersections H and D, which are the centers, and H A and D A the radii respectively of the required tangential circles.

68.—FIG. 2.—**Problem.**—*To construct a circle of a given radius that touches a given circle and a given line.*

Solution.—Let C be the given circle. M N the given line, and R S the given radius of the required circle.

Draw with distance R S the line R′ O parallel to M N. With C A = C B+R S = R B as radius and center C, cut line R′ O in point A, which is the center, and A B = R′ S, the radius of the tangential circle.

69.—FIG. 3.—**Problem.**—*Within a given triangle to inscribe a circle.*

Solution.—Let A B C be the given triangle. Bisect two angles, A and C, by A D and C D, which intersect in D. Draw the perpendicular D E, which is the radius, and D is the center for the inscribed circle.

70.—FIG. 4.—**Problem.**—*To circumscribe about a given triangle a circle.*

Solution.—Let B A D be the given triangle. Bisect two of the sides by perpendiculars, which intersect in the center of the required circle.

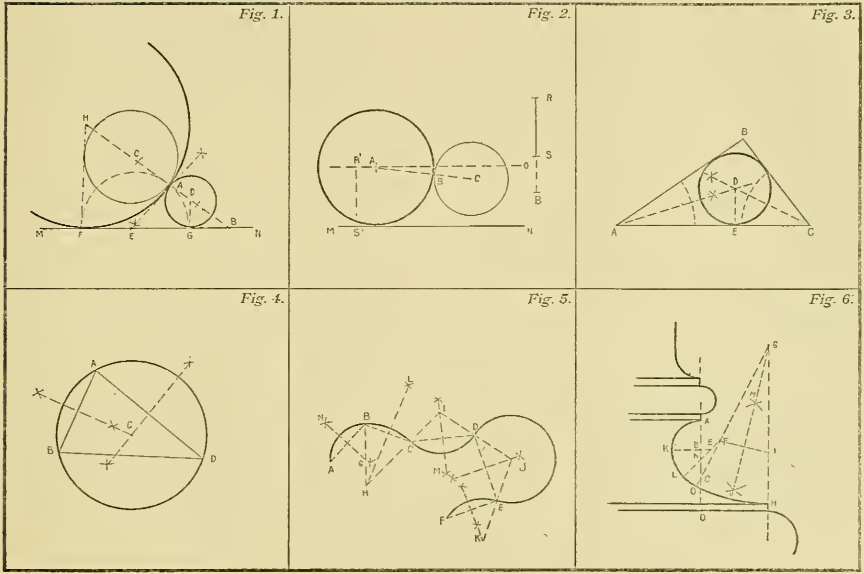
71.—FIG. 5.—**Problem.**—*To connect any number of points by a regular curve.*

Solution.—Let A B C D E, etc., be the given points. Draw lines A B, B C, C D, etc., and bisect each by a perpendicular. Take an arbitrary point G at the bisection line G N as a center, and with G A as a radius draw the arc A B; draw then B G H, a line to intersect the bisecting perpendicular of B C in H, the center, and H B the radius of the arc B C; I is the center, radius I C for arc C D, etc. Complete the required curve to point F.

72.—FIG. 6.—**Problem**.—*To construct a curve to the base of an Ionic column.*

Solution.—Let A D and D H be the given dimensions. Trisect A D and draw in B (1st 3d) a perpendicular, K B E; B A is the radius and B the center of quadrant A K. Make B E, and E F = B N = $\frac{1}{2}$ B A and draw F E N L; E is the center, E K the radius for arc K L. Erect at H a perpendicular, H G, indefinite, at which make H I = L F, and draw and bisect F I by the perpendicular M J, which produced will give the intersection point G; draw line G F O. With F as center, F L as radius, describe arc L O; with G as center, G O as radius, the arc O H.

Plate 17.



Hanstein's Constructive Drawing.

73.—FIG. 1.—**Problem.**—*To construct three tangential circles when their radii are given.*

Solution.—Let A, B and C be the given radii. Draw line G F E = A + B. Describe circle G with radius G F = A, and circle E with radius E F = B. With G as center, and A + C as radius, E as center, B + C as radius, draw arcs intersecting at H. H I is the radius and H the center for the third required tangential circle.

74.—FIG. 2.—**Problem.**—*To construct three tangential circles when the three centers are given.*

Solution.—Let A B C be the given centers. Construct the triangle A B C. Make C D = C B, A E = A B, and bisect D E in point F. Describe the required circles from points C, A and B, as centers, with radii C F, A F and B H.

75.—FIGS. 3 and 4.—**Problem.**—*To construct tangential circles within a given angle.*

Solution.—Let A B C be the given angle, which is bisected by A D. Draw a perpendicular line D C at an arbitrary point D to form angle D C A, which is bisected by C E. The intersection of A D and C E is point E; with E as center, and with the radius E D describe the tangential circle D F. Perpendicular to A D, at point F, draw F G, and parallel with C E, G H. H is the center, H F the radius for the next circle, etc., etc.

76.—FIG. 4.—*Solution* 2.—Bisect the angle B A C by A D and at an arbitrary point, E, erect the perpendicular E F. Make F H = E F and draw perpendicular to A B at H, H I. I is the center, I E the radius of the circle E H J. Repeat this construction by making L M = L J, etc., etc.

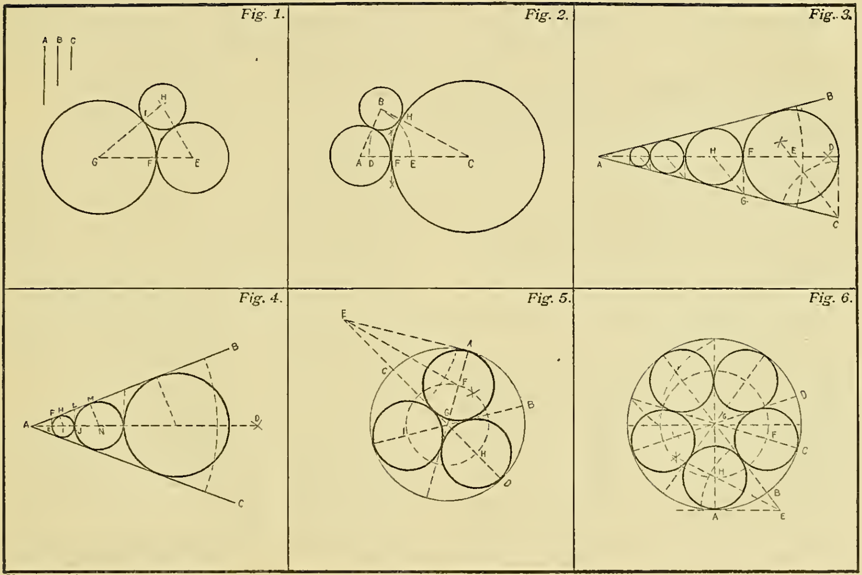
77.—FIGS. 5 and 6.—**Problem.**—*To construct any number of equal tangential circles within a given circle.*

Solution.—Let C A B D be the given circle. Divide the circle into double the number of equal parts as you intend to draw circles therein; for 3 circles into 6, for 5 circles into 10 equal parts.

Construct at an intersection of diameter and circumference point A a tangent to intersect the produced adjoining diameter in E. Bisect angle A E G by E F; F is the center, F A the radius for one required circle. With center G of the given circle and radius G F draw circle F I H, to obtain I and H, the centers of the required remaining tangential circles.

Problem Fig. 6 is solved in a similar manner.

Plate 18.



Hanstein's Constructive Drawing.

TANGENTIAL CIRCLES.

78.—Fig. 1.—**Problem.**—*To divide the surface of a circle into three equivalent parts bounded by semi-circles.*

Solution.—Let C be the given circle. Divide the diameter D A into 6 equal parts, and describe with 1 and 5 as centers, 1 D as radius, the semicircles 2 D and 4 A, with 2 and 4 as centers, and 2 D as radius, the semicircles D 4 and 2 A; D 4 A 2 = $\frac{1}{3}$ of area of circle.

79.—Fig. 2.—**Problem.**—*To construct a rosette of four units within a given circle.*

Solution.—Let A B be the diameter of the given circle. Draw four equal tangential circles within the given circle (See Figs. 5 and 6, Plate 18) and connect their centers by the lines F E, E D, D H, and H F, which at I, I′, I″, and I‴ pass through their points of contact.

Concentric arcs may be added to indicate material.

80.—Fig. 3.—**Problem.**—*To construct three tangential circles within a semicircle.*

Solution.—Let A D B be the given semi-circle. Divide the radius C D into 4 equal parts, erect at point 1, E F perpendicular to C D, and describe with C as center, and radius C 3, the arc E 3 F. Point 2 is center, 2 D the radius to circle C D, and E and F are the centers to the required tangential remaining circles.

A and B are the centers, A B the radius to arcs A G and G B, which form a Gothic arch.

81—Fig. 4.—**Problem.**—*To construct two semicircles and three circles tangential within a given semicircle.*

Solution.—Let A 3 B be the given semicircle. Divide radius C 3 into 3, the diameter A B into 4 equal parts; erect at E and F, E H and F G perpendicular to A B, and at 2, H G perpendicular to C 3. E F are the centers, E A the radius to semicircles A C and C B; 2 and 4 centers, 2—3 the radius to circles 2—4 and 4—3, and H and G the centers for the required remaining tangential circles.

GOTHIC AND PERSIAN ARCHES.

82.—Fig. 5.—**Problem** —*To construct a Gothic arch on an equilateral triangle.* (*Inscribe a tangential circle.*)

Solution.—Let A C B be the equilateral triangle. Describe with B and A as centers, and radius A B the arcs A C and C B; A E C H B is the required Gothic arch.

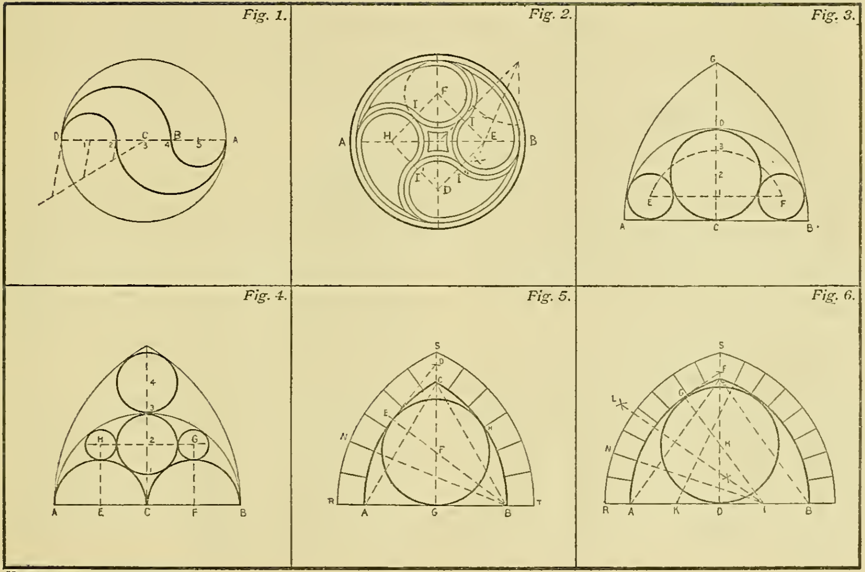
Center F of a tangential circle in this arch is found by making D G = B A, D E = B G, and drawing E B, intersecting C G, in F; the center F and radius F G give the required tangential circle.

Remark.—When A R represents the thickness of the stone required in work, the arcs R S and S T are concentric with A C and C B. The lines representing the joints of stones, as N B (voussoir-lines), are radii in the corresponding sector.

83.—Fig. 6.—**Problem.**—*To construct a Gothic arch when span and altitude are given.*

Solution.—Let A B be the given span and D E the altitude. Construct an isosceles triangle, A E B, with A B as base and D E as altitude; bisect A E by the perpendicular L I, which intersects span A B in I. I and K are the centers, I A the radius to arcs A E and E B. Make D F = A I, and F G = D I, and draw G I, intersecting D E, in H, the center, H D, the radius to the tangential circle in arch A E B.

Plate 19.



Hanstein's Constructive Drawing.

84.—FIG. 1.—*Solution* 2.—Let A B be the span and C D the given altitude. Construct an isosceles triangle, A D B, in which the base = A B, the altitude = C D. Bisect A D by the perpendicular L I, intersecting the produced span in I; I and J are the centers, I A is the radius to arcs A D and B D. A D B is the required Gothic arch. To find center H for the inscribed circle, make C E = A I, E F = C I and draw F H I.

85.—FIG. 2.—**Problem.**—*To construct a Gothic arch (wood or stone) with application of previous constructions for its inside ornamentation.*

Remark.—This problem is intended as a review of former constructions, and should be drawn not less than three times the size of Fig. 2, to avoid inaccurate work by crowded lines.

86.—FIG. 3.—**Problem.**—*To construct a Persian arch about an equilateral triangle.*

Solution.—Let A D B be the equilateral triangle. Divide A D into 3 equal parts and draw through point 2, parallel with D B, G 2 E, intersecting G H in G and A B in E. Make D H = D G, and draw H F parallel to D A. E and F are centers to arcs A 2 and B I, and G and H the centers to arcs 2 D and I D; A 2 D I B is the required Persian arch.

87.—FIG. 4.—**Problem.**—*To construct a Persian arch when* A B, *the span, and* C D, *the altitude, are given.*

Solution.—Construct with span A B as base, and with altitude C D the isosceles triangle A D B. Trisect A D and erect in point 1 the perpendicular 1 E; draw E 2 G, intersecting G H (parallel to A B) in G. Continue as in the previons construction and obtain the required Persian arch.

EGG-LINES.

88.—FIG. 5.—**Problem.**—*To construct an egg-line on a given circle.*

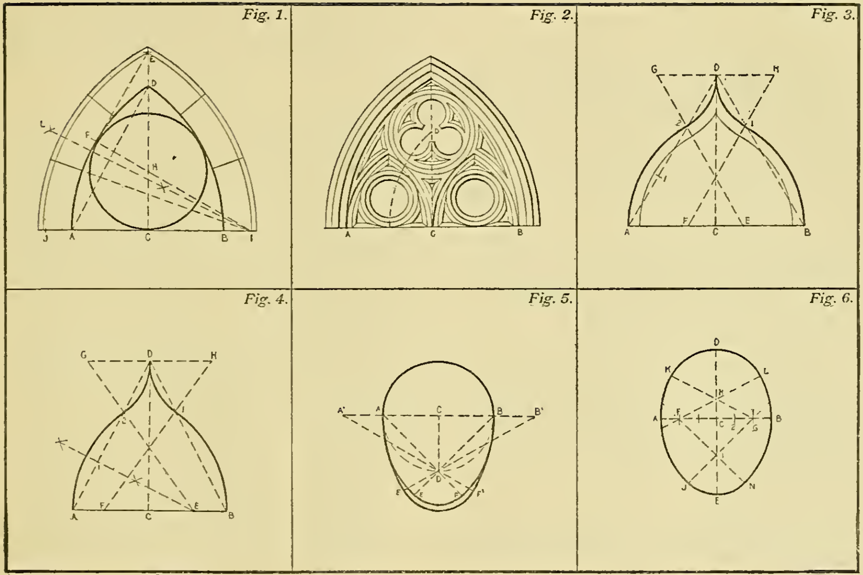
Solution.—Let C be the given circle. Draw perpendicular diameters A B and C D, also lines B D E and A D F; B and A are centers, radius = A B to arcs A E and B F, and D center to arc E F; A B F E is the required egg-line. To obtain a more elongated shape of an egg-line, place centers A′ B′ further out, but equidistant from C, and describe arcs A E′ and B F′, and with D as center arc E′ F′.

89.—FIG. 6.—**Problem.**—*To construct an egg-line when the short axis is given.*

Remark.—The longest line possible to be drawn in the egg-line is called its long axis, and the greatest width perpendicular to it is the short axis.

Solution.—Bisect the given short axis A B by the perpendicular D E, on which make H C $\frac{1}{6}$. C I $\frac{2}{6}$ of A B; C F = C G = $\frac{2}{6}$ of A B; F and G are centers, and F B the radius to arcs L N and K J, H to K L and I to J N; K L N J is the required egg-line.

Plate 20.



OVALS.

90.—FIG. 1.—**Problem.**—*To construct an oval or lens-line at adjoining equal squares.*

Definition.—An oval is an elongated *endless* curve consisting of symmetric arcs. The longest possible line drawn in an oval is called its *long axis*, and the greatest width perpendicular to it is called its *short axis*. Both axes divide the oval into symmetric parts.

Solution.—Let A F G C and F G D B be the given squares. Draw the diagonals F C and A G, intersecting in H, and F D and B C, intersecting in I; G and F are centers, G A, the radius to arcs A B and C D, H and I the centers to arcs A C and B D.

91.—FIG. 2.—**Problem.**—*To construct an oval at a given circle.*

Solution.—Let A B F G be the given circle. Construct two perpendicular diameters, A F and B G, and draw A B D, A G I, F B E and F G H; F and A are the centers, radius F A to arcs E A H and D F I; B and G are the centers, radius B D to arcs E D and H I; E H I D is the required oval.

92.—FIG. 3.—**Problem.**—*To construct an oval, at two equal circles, of which the circumference of one passes through the center of the other.*

Solution.—Let A and C be the given circles, intersecting each other in B and D. Draw from points B and D through centers A and C, lines B A G, B C H, D A F and D C E. D and B are the centers, radius D F to arcs F E, and G H. F E H G is the required oval.

93.—FIG. 4.—**Problem.**—*To construct an oval when its long and short axes are given.*

Solution.—Let A B and C D, bisecting perpendicularly, be the long and short axis respectively. Draw C B and the quadrant C K from center E. Make C N = K B and bisect N B by the perpendicular O L H; I E = E H, and E J = L E, and draw H J P, I J R and I L S. J and L are the centers, J A the radius to arcs P R and O S, and H and I the centers to arcs P O and R S; P O S R is the required oval.

ARCHES.

94.—FIGS. 5 and 6.—**Problem.**—*To construct an arch, its span and altitude being given.*

Solution.—Let A B be the given span, and C D, the perpendicular in its bisection point C, the altitude. Construct with ½ A B = A C and C D the rectangle C D E A, and draw diagonal D A. Bisect angles E D A and E A D by F D and F A. From F, perpendicular to A D, draw F H G and make I C = H C; H and I are the centers, with radius H A to arcs A F and B J, and G the center, radius G F to arcs F D J; A F D J B is the required arch.

Remark.—When we assume the thickness of the stone used in the arch as B O, we describe the concentric arcs O N, N E and E L, and divide these into equal parts, except keystone K, to which generally more prominence is given. As in the Gothic arches, the joint lines of the stones are radii in the corresponding sector.

95.—FIG. 6.—*Solution* 2.—Let A B be the given span, and C D the altitude. With E A as a radius shorter than the given altitude, and centers E, D and F, describe the circles E A, D G and F B; draw and bisect E G, intersecting the produced altitude in point H, the center, with radius H I to arc I D L. Complete the required arch A I D L B and add its stone units.

Plate 21.

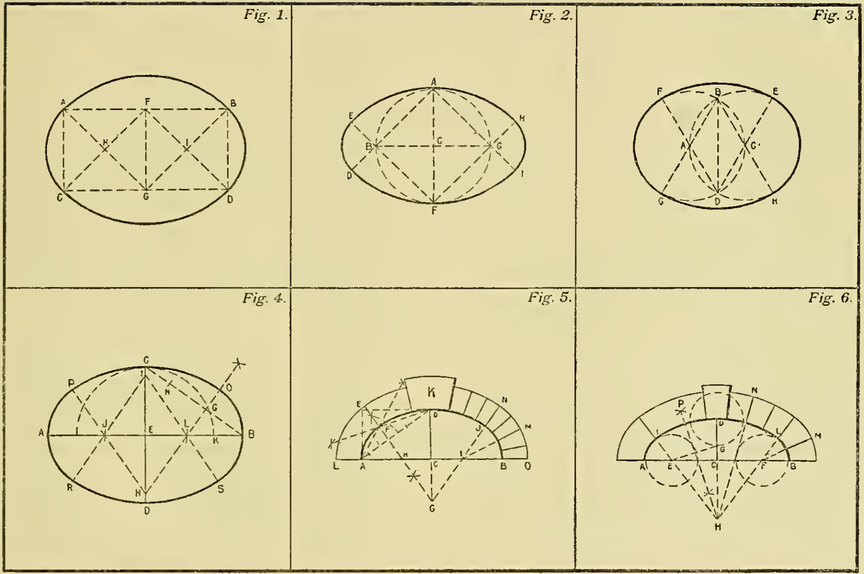


Fig. 1.
A F B H I C G D
Fig. 2.
A E H B C G D I F
Fig. 3.
F B E A C G D H
Fig. 4.
C P O N G A J E L K B R H D S
Fig. 5.
K E N D F J M K L A H C I B O G
Fig. 6.
N P D I L M G A E C F B H

96.—FIG. 1.—*Solution* 3.—Let A B be the span, and C D the altitude. Construct with A C the equilateral triangle A E C, and make C F = C D, and draw D F G. Parallel with E C draw G H I; points H and K are the centers, A H the radius to arcs A G and J B, and I the center, I G the radius to arc G D J. Proceed as in Fig. 6, and complete the required arch and its stone units.

97.—FIG. 2.—*Solution* 4.—Let A B be the span, and C D the altitude. Construct on altitude C D the equilateral triangle D E C, make C F=C A, etc., and proceed and complete as in Fig. 1.

98.—FIG. 4.—**Problem.**—*To construct an elliptic arch when span and altitude are given.*

Solution.—Let A B be the given base, C D the altitude. Produce A B, and with radius C′ D′ = C D = the given altitude describe semicircle J D′ A. Divide J A and span A B into the same number of equal parts, and erect at all division points perpendiculars. With the T square make C D = C′ D′, 2 E′ and 4 E″ = 2 E, 1 G′ and 5 G″ = 1 G, etc., and connect points B G″ E″ D E′ G′ A by a free-hand line, and complete the required elliptic arch.

99.—FIGS. 3 and 5.—**Problem.**—*To construct ascending arches when span and altitude are given.*

Solution.—Let A B be the given span and C B the altitude; draw C A, the ascending line. Make B D (the produced span) = B C, and bisect A D by the perpendicular E G; E is center, E A the radius to quadrant A G, and F the center, F G the radius to quandrant G C. A G C B is the required arch. Complete and add the stone units as in previous constructions.

100.—FIG. 5.—*Solution* 2.—Let A B be the given span, and B D the altitude. Draw D A, the ascending line, and bisect A B by the perpendicular F C; bisect angle F E A by G J, and with J as center, J A as radius, describe arc A F. Draw F J and D H parallel to A B, and with center H, radius H D describe arc F D; A F D B is the required ascending arch. Complete and add stone units as in previous constructions.

Fig. 1.

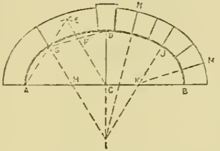


Fig. 2.

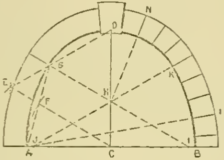


Fig. 3.

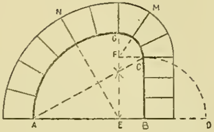


Fig. 4.

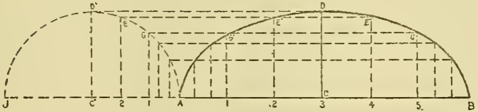
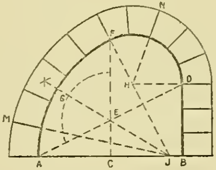


Fig. 5.



101.—Figs. 1 and 1 A.—**Problem.**—*To construct an Ionic spiral when the altitude is given.*

Solution.—Let A B be the given altitude. Divide A B into 16 equal parts. The center of the spiral eye is situated in the 9th part from B, and its radius = $\frac{1}{16}$ of A B.

Fig. 1 A.—*Remark.*—To explain division and subdivision, the eye of the spiral in double size is represented in Fig. 1 A. It is advisable to execute Fig. 1, Plate 23 and Fig. 1, Plate 24 in as large a scale as possible, to facilitate an accurate division and subdivision.

Draw two perpendicular diameters, D G and F H, and inscribe the square D F G H; inscribe in this the square 1, 2, 3 and 4. Divide the diagonals 1 3 and 2 4 into 6 equal parts and draw the squares 5, 6, 7 and 8, and 9, 10, 11 and 12.

The center of the first quadrant is point 12, the radius 12 D, describe D I; 11 the center, 11 I the radius to quadrant I K, etc., and go back as the numbers indicate, 10, 9, 8, 7, 6, 5, 4, 3, 2, until the center of the last quadrant is point 1, the radius 1 J to quadrant J B. To obtain the second curve, we trisect the distances 12 C, 11 C, 10 C, 9 C; also 5, 9 — 6, 10 — 7, 11, etc., etc., and with the center in the first 3d from 12 towards C, draw the first quadrant; first 3d, from 11 towards C, as center the next quadrant, etc., etc., and complete the curve in the same order, locating the centers at diagonals in the first 3d from the original division towards C.

102.—Fig. 2.—**Problem.**—*To construct an elliptic ascending arch when span and altitude are given.*

Solution.—Let A B be the span and B C the altitude. Draw C A, the ascending line, and describe on A B as diameter, a semicircle, A N O B. Divide the diameter into any number of equal parts (6) and erect in each division perpendiculars, at which we make 2′ N′ = 2 N, 4′ P = 4 P and N′ R′ = N R and connect R′ O′ P′ N′, etc., by a free-hand line, which is the required arch.

Remark.—This curve is also applied at the base of the Ionic column, as Fig. 6, Plate 9.

103.—Fig. 3.—**Problem.**—*To construct an elliptic ascending arch when span, its ascending and mean altitudes are given.*

Solution.—Let A B be the given span, B C the ascending and E F the mean altitude. With the mean altitude E F = E′ F′ describe the quadrant F′ H A E′; divide radius E′ A in 3 and subdivide the last 3d into 3 equal parts. Divide the span into the same number of proportional parts and erect perpendiculars. Transfer the altitudes of F′ H J, etc., to the perpendicular A D, and draw lines parallel with the ascending line A C, to obtain the points of intersection J′ J″, H′ H″, F, etc., which points, connected by a free-hand line, will give the required arch, C J″ H″ F H′ J′ A.

Fig. 1.

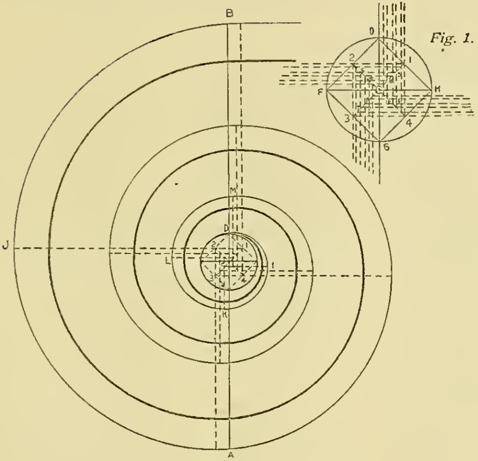


Fig. 1. A

Fig. 2.

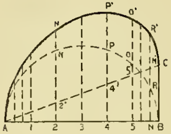
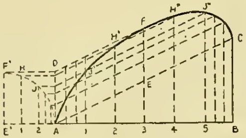


Fig. 3.



104.—FIGS. 1 and 1A.—*Solution.*—Let A B be the given altitude of the spiral, which is divided into 14 equal parts. Point C, the center of the eye of the spiral, is located at the 8th part from B; its radius $\frac{1}{14}$ of A B. Construct the square D F E H (see Fig. 8), and inscribe the square 1, 2, 3 and 4; bisect C 1, C 2, C 3 and C 4; bisect again C 5, C 6, C 7 and C 8 and draw the squares 5, 6, 7, 8 and 9, 10, 11, 12. Describe, first, the quadrant D I, from center 12 and radius 12 D, 2nd, quadrant I K, from center 11, with radius 11 I, etc., as operated in Fig. 6, until point 1 is the center, I J the radius of the last quadrant J B of the third revolution. The subdivision for the centers of the second curve is as follows:

Bisect C 12, C 11, C 10 and C 9, and bisect also 12, 8 — 11, 7 — 10, 6 and 9, 5, and make these bisection points vertices of squares parallel to the square 1, 2, 3 and 4. Divide further the lines 1, 5 — 2, 6 — 3, 7 and 4, 8 into 4 equal parts, and construct a square parallel to the vertices located in the first 4th from points 1, 2, 3 and 4 towards C. The vertices of the squares of these subdivisions are the centers for quadrants of the second curve, which quadrants are described as in Fig. 6.

SPIRALS.

105.—FIG. 2.—**Problem.**—*To construct a spiral with semi-circles when the spiral "eye" is given.*

Solution.—Let C, a small circle, be the given spiral eye. Draw and produce a horizontal diameter, M A B N. With A as center, A B as radius, describe the semi-circle B O; C as center, C O as radius, semi-circle O P; A as center, A P as radius, semi-circle P R, etc. Curve B O P R, etc., is the required spiral.

106.—FIG. 3.—**Problem.**—*To construct the evolute of a given triangle.*

Solution.—Let A B C be the given triangle. Produce C B, B A and A C; B is the center, B A the radius to arc A N; C the center, C N the radius to arc N O; A the center, radius A O to arc O P, etc., etc. Curve A N O P, etc., is the required evolute.

Fig. 1.

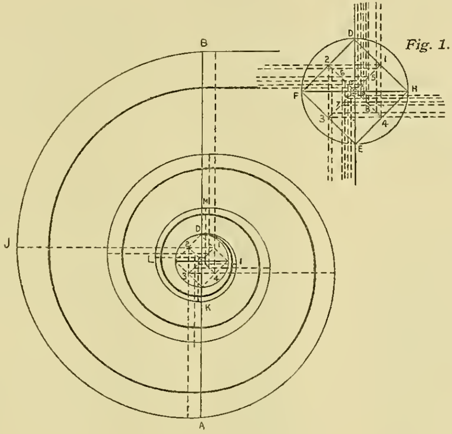


Fig. 1. A

Fig. 2.

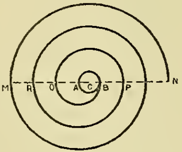
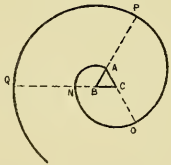


Fig. 3.



Definition.—An archimedean spiral is a curve in a plane generated by a point whose distance from a centre of rotation increases uniformly.

Cams are arrangements in mechanics by which a rotary motion is converted into a reciprocating action, they are constructed by archimedean spirals.

Remark.—The following curves, used principally in mechanics and architecture, should be executed by free-hand lines before the student attempts to use a curve rule.

107.—FIG. 1.—**Problem.**—*To construct a cam-line of 1⅓ revolutions when the distance C C' between revolutions is given.*

Solution.—With 8 equal parts, 6 of which are equal to the given distance C C', describe the circle 8 A B D E F, which is divided into 6 equal parts by diameters. Describe circles with C as center, radius C 1, to intersect diameter B F in B'; with radius C 2 to intersect D 8 in D'; C 3 to intersect E A in E', etc.; connect points C B' D' E' F' 5 C' H D by a free-hand line, to complete the required cam-line.

108.—FIG. 2.—**Problem.**—*To construct a heart-shaped cam when the altitude is given.*

Remark.—Heart-shaped cams are made to convert half of a revolution into forward motion, the other half of the revolution into backward motion. (Piston-rods for pumps, etc.)

Solution.—Let C 8 be the given altitude, which is divided into 8 equal parts and is the radius, C the center of the circle, divided by diameters into 16 equal parts. With center C, radius C 1, describe circle to intersect radii C A and C G in A and A', with C 2 as radius to cut radii B C and J C in B' and B'', with C 3 to cut radii C C and K C in C' and C'', etc. Connect C A' B' C' D' I E H C'' B'' A'' C by a free-hand line and complete the required heart-shaped cam.

109.—FIG. 3.—**Problem.**—*To construct a cam in 4 equal divisions, to raise a lever in the first ¼ of its revolution, equal to the altitude* B D, *to remain stationary the second ¼, to descend its first position the third ¼. and remain stationary the last ¼ of its revolution.*

Solution.—Let B D be the given altitude, B A an arbitrary tance from the *hub*, and C the center of the cam. Describe with C D, center C, the circle D H D' 4 and divide it into quadrants, two opposite ones into 4 equal parts again, by diameters. N 4 = B D = the given altitude is also divided into 4 equal parts, 1, 2, 3 and 4, and with radius C 1 draw arcs 1 G', G, with radius C 2, 2 F' F, wit radius C 3, 3 E' E; connect N G' F' E' D' and the symmetric points B G F E H by a free-hand line and complete the required cam.

110.—FIG. 4.—**Problem.** *To construct a cam in three equal divisions, which in one revolution shall lift a lever* = A 4 *in the first 3d, shall remain stationary the second 3d, and shall rise again the third 3d an altitude* = 4 B *and make a sudden escape at* B, *to renew its motion in the second revolution.*

Solution.—Let the two inner circles be shaft and hub circumferences, A 4 the altitude of the first incline, 4 B the altitude of the second incline (the third division).

Remark.—This construction, in applying the principles of Figs. 1, 2 and 3, will not present any difficulty to the student, and can now be solved without the assistance of a teacher.

CONIC SECTIONS. ELLIPSE, PARABOLA AND HYPERBOLA.

111.—FIG. 5.—Three curves, which we obtain by sectional planes through a circular cone and cylinder, are of the greatest importance in technical work; the *ellipse, parabola and hyperbola.* A sectional plane through the cylinder or circular cone in an oblique direction, as U V or M N, respectively, creates the *ellipse.* A sectional plane S T, parallel to the side C B of the circular cone, creates the *parabola.* A sectional plane Q R, parallel to the axis of the circular cone, creates the *hyperbola.*

Definition.—An ellipse is a closed curve; the sum of the distances of each point in this curve from two fixed points within, called foci, is equal to the long axis. The ellipse has two axes, the major and minor, bisecting each other perpendicularly and dividing the ellipse as well as its surface into two symmetric parts.

112.—FIG. 6.—**Problem.**—*To construct an ellipse when major axis (transversant) and minor axis (conjugant) are given.*

Solution.—Let A B be the major, C D the minor axis. When a radius ½ A B = A M and center C draw arc and intersections with A B, points F and F', the foci; divide F M arbitrarily into parts, increasing in length towards M, and with F and F' as centers, B 4 as radius, describe arcs E G and E' G'; with F and F' as centers, A 4 as radius, draw intersections at E and G and at E' and G'. Points E E' G G' are situated at the circumference of the ellipse. Operate with points 3, 2 and 1 in the same manner, and we obtain by each operation 4 points, which lie at the circumference of the ellipse, as with point 2, e. g., by which we locate points H J H' J'. Connecting these points by a free hand line, we obtain C E H A J G, etc., the required ellipse.

Fig. 1.

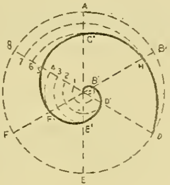


Fig. 2.

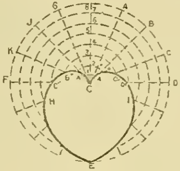


Fig. 3.

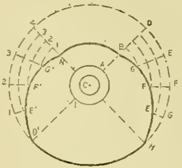


Fig. 4.

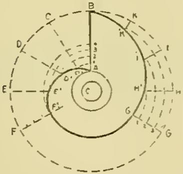


Fig. 5.

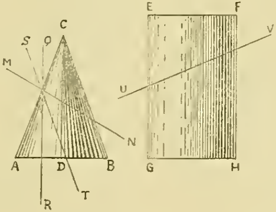
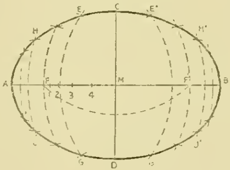


Fig. 6.



113.—FIG. 1.—**Problem.**—*To construct a tangent to an ellipse when the point of contact is given.*

Solution.—Let A C B D be the ellipse and G the point of contact. Describe from G as center, with radius G F, the arc F N, and draw and produce line F' G, intersecting arc F N in N; bisect angle N G F by I J, which is the required tangent.

Remark.—In elliptic arches, executed in cut stone, the joints are perpendiculars (as P G) to tangents, having the unit divisions as points of contact.

114.—FIG 1.—**Problem.**—*From an exterior point to construct a tangent to an ellipse.*

Solution.—Let H be the given exterior point. With H as center, H F' as radius, describe arc F' O; with A B as radius, and F as center, intersect arc F' O in O. Bisect arc F' O by L H, which is the required tangent.

115.—FIG. 2.—**Problem.**—*To construct an ellipse when both axes are given. (Practical solution.)*

Solution 1.—Let A B and C D be the given axes. Find the foci (112) and place in F, F' and C pins, around which tie a linen thread to form the triangle F C F'. Take away the pin at C and place the pencil point in the triangle, by stretching the thread gently and forming a vertex of the triangle; draw the curve, which will be the required ellipse.

Solution 2.—A B and C D are the given axes. Take O P, a straight edge or a slip of paper, at which make A' M' = A M = ½ A B and A' C' = C M = ½ C D. Guide the straight edge to have point C' follow the major axis, and M' the minor axis, then will point A' describe the circumference of the required ellipse. Locate the position of point A' during this operation by pencil marks, which, connected, will give the ellipse.

Remark.—Place in points C' and M' pins, in point A' a pencil point, and let these pins slide in grooves in the place of the axes; we have an instrument called a trammel or ellipsograph, with which we are able to draw any ellipse by arranging points A' C' and M' in the required proportions.

116.—FIG. 3.—**Problem.**—*To construct an ellipse by intersecting lines.*

Solution.—Let A B and C D be the given axis, and construct with these lines the rectangle E F G H; divide A B and E G into the same number of equal parts and number as in the diagram. Draw lines D 1 P, D 2 O and D 3 N, intersecting the lines C 1, C 2 and C 3 at P, O and N, etc., which points, connected by a free-hand line, will be the required ellipse.

117.—FIG. 4.—**Problem.**—*To construct an elliptic curve in an oblique parallelogram.*

Solution.—Let E F G H be the parallelogram. Draw axes A B and C D bisecting opposite sides, and divide C M and E C into the same number of equal parts; proceed as in the previous construction and draw C P O N A, etc., the required ellipse.

118.—FIG. 5 A.—**Problem.**—*To construct an ellipse by intersections of lines.*

Solution.—With A B and C D, the given axes, coustruct the rectangle E F H G; divide E O and A E in the same number of equal parts (4) and number as shown in the diagram. Draw lines 1 A, 2 3, 3 2, and C 1, and connect their intersections T S R, etc., by a free-hand line to complete C T S R A, etc., the required ellipse.

119.—FIG. 5 B.—**Problem.**—*To construct an ellipse by its tangents.*

Solution.—Draw and divide C B into any number of equal parts (4): 1, 2, 3 and 4, through which parallel with C D draw P P', O O' and L L'; draw also E 1 I, E 2 K and E 3 N and lines L N, O K and P I, which are the tangents to the required ellipse. Draw the ellipse by a free-hand line.

120.—FIG. 6.—**Problem.**—*To construct an ellipse by the differences of two circles.*

Solution.—Let B A and C D be the given axes. Describe with B A and C D as diameters concentric circles with center M. Divide both circles into 12 equal parts by the diameters 10, 4—11, 5—1, 7—2, 8 and 3, 9. Draw lines 7, 5—8, 4—10, 2 and 11, 1, and from the intersection points E F G and H the perpendiculars to 10, 2—11, 1—7, 5 and 8, 4, which will give points N O A P R D, etc., at the circumference of the required ellipse.

Plate 26.

Fig. 1.

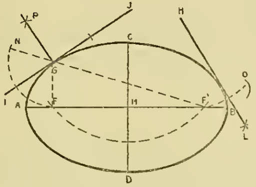


Fig. 2.

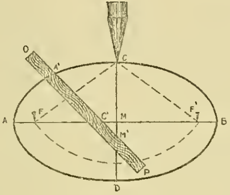


Fig. 3.

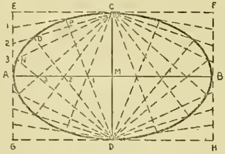


Fig. 4.

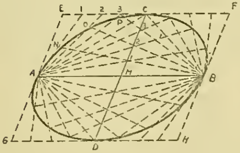


Fig. 5.

Fig. 5.

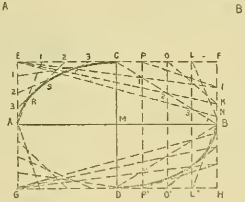
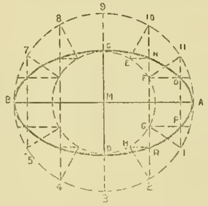


Fig. 6.



Hanstein's Constructive Drawing.

121.—Fig. 1.—**Problem.**—*To construct a parabola when the axis and the base are given.*

Definition.—The parabola is a curve in which the distance of any point from an outside right line (directrix) is equal to the distance of this point from a fixed point within, called focus. A line bisected perpendicularly by the axis at its terminus and intersecting the curve is called the base, and a parallel with it, through the focus, the parameter of the parabola.

Solution.—Let A P be the axis and L K the given base. Bisect L P = ½ the base L K in J, and draw J A. In J erect a perpendicular to J A, J R intersecting the produced axis in R; transfer P R to left and right of point A, to obtain point F, the focus, and point O, through which draw M N, the directrix, perpendicular to the axis O P. Divide A P into arbitrary parts, 1, 2, 3, 4, etc., in which erect perpendiculars, and with F as center, O 1 as radius, cut the perpendicular 1 in B and B'; with O 2 as radius, the same center, cut the perpendicular 2 in C and C'; with O 3 as radius cut perpendicular 3 in D and D', etc., and connect the obtained points L E' B' A B E K by a free-hand line, which is the required parabola.

122.—Fig. 2.—**Problem.**—*To construct a tangent to a parabola when the point of contact is given.*

Solution.—Let L R K be the given parabola, O P the axis, M N the directrix, and A the point of contact. With A as center, A F as radius, draw arc F B and A B perpendicular to M N. Bisect arc F B by line S G, which is the required tangent.

Problem.—*To construct a tangent to a parabola from an exterior point,* E.

Solution.—With E as center, and E F as radius, draw arc F D and erect at D a perpendicular to M N, intersecting the parabola in H, the point of contact; or bisect arc D F by line T E, which is the required tangent.

123.—Fig. 3.—**Problem.**—*To construct a parabola when two symmetric tangents are given.*

Solution.—Let B E = A E be the given tangents. Divide E B and A E into equal parts and number as shown in the diagram. Draw lines 7–7, 6-6, 5-5, 4–4, etc., which are the tangents of the parabola. A free-hand curve tangential to these tangents is the required parabola.

124.—Fig. 4.—**Problem.**—*To construct a parabola when the axis and the base are given or the rectangle drawn with these lines.*

Solution.—Let A B 6 J be the given rectangle. Divide ½ 6 J = D 6 and B 6 into 6 equal parts, respectively; number as in the diagram, and draw parallel to the axis D C lines through 1, 2, 3, 4, 5. Draw also lines 5 D, 4 D, 3 D, 2 D and 1 D, intersecting with the horizontals in points I H G E F D, etc., which points, connected by a free-hand line, furnish the required parabola.

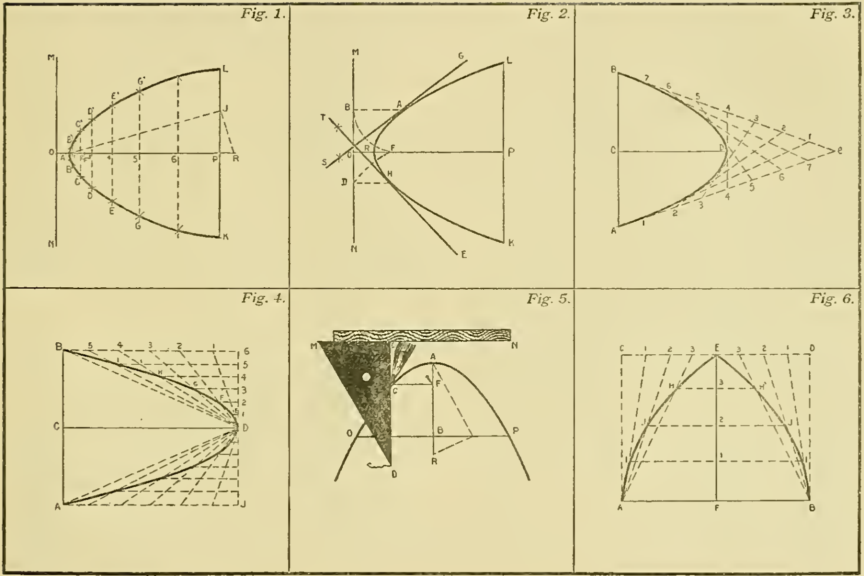
125.—Fig. 5.—**Problem.**—*To construct a parabola practically when base,* O P, *and axis,* A B, *are given.*

Solution.—Locate the focus F and the directrix M N and place a straight edge firmly coinciding with it. Fasten a thread to a pin placed in F and pass it around a pin in A to a point D of the set square, when its side C D coincides with axis A B. Remove the pin in A and hold the pencil to stretch the thread gently, touching C D constantly, shift the set square to the left. The pencil point will describe the required parabola on the drawing paper.

126.—Fig. 6.—**Problem.**—*To construct a Gothic arch by parabolas.*

Solution.—Let A B be the span and F E the altitude of the arch. Construct the rectangle C D B A, divide C D into 8 and E F into 4 equal parts and number as the diagram. Draw lines 1 A, 2 A, 3 A and parallel to span I 1 I', J 2 J', and H 3 H'. The points of intersection, A I J H E H' J' I' B, connected by a free-hand line, complete the arch.

Plate 27.



Hanstein's Constructive Drawing.

127.— Fig. 1.— **Problem.**— *To construct hyperbolas when the vertices and foci are given.*

Definition.—The hyperbolas are curves; the difference of distances of each point to the foci is equal to an invariable line, the axis.

Solution.—Place on line M N, A and B the vertices, and F and F′ the foci equidistant from O. From F′ towards M mark arbitrary divisions and number as in diagram. With radius B 1, center F, — radius A 1 and center F′ draw intersecting arcs at C and C′; radius B 2, center F and radius A 2 and center F′ draw intersecting arcs at D′ and D, etc. Connect G′ E′ D′ C′ A C D F G by a free-hand line, to complete the required hyperbola. To obtain the second curve, operate symmetrically.

128.—Fig. 2.—**Problem.**—*To construct a tangent to a hyperbola when point of contact*, P, *is given.*

Solution.—Draw line P F, and with radius P F′ and center P the arc F′ D. Bisect F′ D by the line T U, which is the required tangent to the hyperbola.

Remark.—The stone joints in hyperbolical arches are the perpendiculars to tangents at the point of contact.

Problem.—*From an exterior point*, R, *to construct a tangent to the hyperbola.*

Solution.—With R as center and radius R F draw arc F N; with F′ as center and radius A B cut arc F N in N and bisect F N by S R, which is the required tangent to the hyperbola.

129.— Fig. 3.— **Problem**.— *To construct hyperbolas when axis* A B *is given; to find foci and draw the asymptotes.*

Asymptotes are right lines to which the branches of the hyperbolas do approach when produced, but do not touch.

Solution.— Construct the square E D C G with C D = A B, which the axis divides into two equal rectangles. Draw and produce the diagonals M N and O P, which are the required asymptotes. With O as center, O G as radius, draw arcs G F′ and C F. With F and F′, the required foci, draw the hyperbolas, as in Fig. 7.

EVOLUTE.

130.—Fig. 4.—**Problem.**—*To construct an evolute at a given circle.*

Definition.—An evolute is a curve made by the end of a string unwinding from a cylinder.

Solution.—Let C be the given circle (the section of a cylinder). Divide the circumference into a number of equal parts (12) and draw the diameters and tangents 1 A′, 2 A″, 3 A‴, 4 A^4, etc. With center 1 and radius 1 A describe arc A A′; center 2, radius 2 A′, the arc A′ A″; center 3, radius 3 A″, the arc A″ A‴, etc.; curve A, A′, A″, A‴ is the required evolute.

Fig. 1.

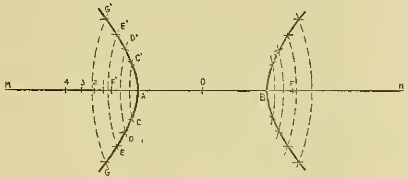


Fig. 2.

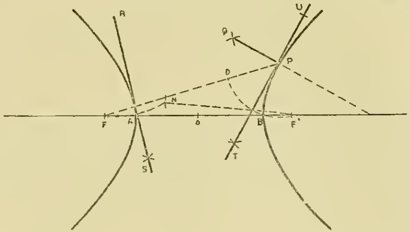


Fig. 3.

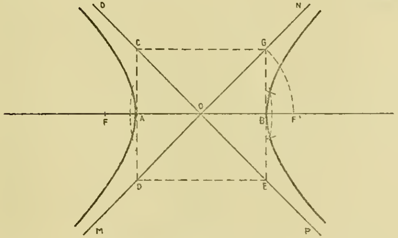
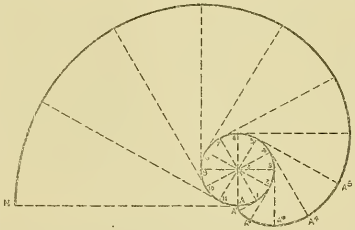


Fig. 4.



Hanstein's Constructive Drawing.

131.—FIG. 1.—**Problem.**—*To construct a cycloid when the generating point A is given at the circumference of the circle.*

Definition.—A cycloid is a curve generated by a point at the circumference of a circle, making one revolution in rolling on a straight line. The curve generated, when the circle rolls on the outside circumference of another circle, is the *epicycloid*, and when the circle rolls on the inside circumference of another circle, the *hypocycloid.*

Solution 1.—Let C be the rolling circle, tangent A B its rectified circumference and A the generating point. Divide the circle C and line A B into the same number of equal parts (12) and number as in diagram. Pass horizontals through points 1, 2, 3, etc., of the rolling circle and erect perpendiculars at A B in points 1, 2, 3, etc. With points C′, C″, C^3, C^4 as centers, C A as a radius, describe circles 1 A′, 2 A″, 3 A‴, 4 A^4, etc., which points connected give the required cycloid.

132.—FIG. 2.—*Solution* 2.—Follow the operations of the previous construction. Draw the circle C^6 6, also chords 6 I, 6 H, 6 G, 6 F, 6 E and their symmetric chords. Parallel to 6 E draw E′ 7 K, to 6 F, F′ 8 L′, to 6 G, G′ 9 M′, to 6 H, H′ 10 N′ and to 6 I, I 11 O′. O′ is the center, O′ B the radius to arc B I′; N′ the center, N′ H′ the radius to H′ G′; M′ the center to G′ F′, L the center to F′ E′ and K to E′ D E, etc. Complete the construction symmetrically to the left of axis D K. The curve of B I′ H′ G′, etc., is the required cycloid.

When a cycloidal arch is executed in stone, the radii of the pertaining arcs are the joints of the units.

133.—FIG. 3.—**Problem.**—*To construct a cycloid when the point generating the curve is situated at a greater radius than the rolling circle.*

Solution.—Let C G be the rolling circle, G 12 its rectified circumference and A the generating point. Describe with C A from C a concentric circle and proceed in this construction as in Fig. 1. Pass horizontals through the divisions of the greater circle and describe with radius C A and centers C′, C^2, C^3, etc., the circles B A′, C A″, D A‴, etc. Points A, A′, A″, A‴, etc., connected by a curve, are the required cycloid.

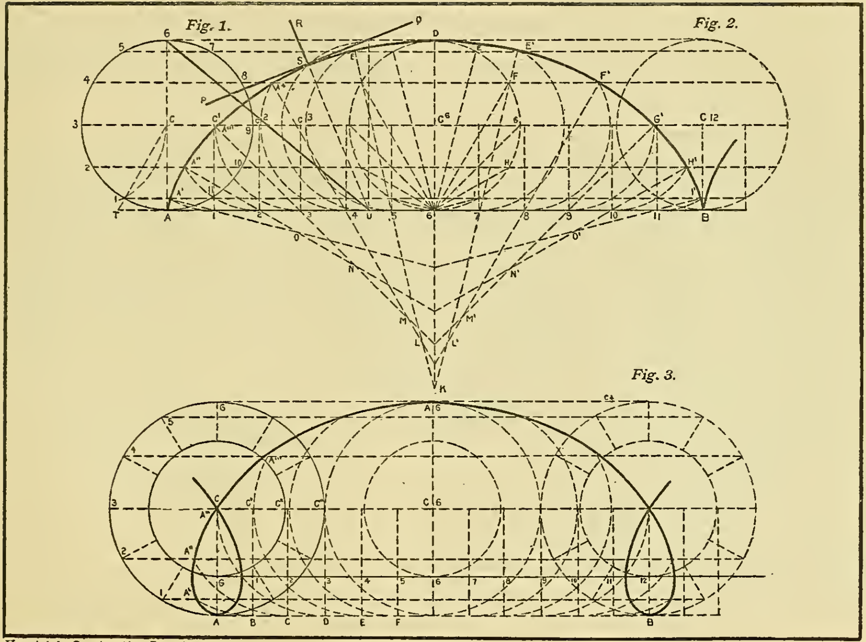


Fig. 1.
Fig. 2.
Fig. 3.

134.—FIG. 1.—**Problem.**—*To construct an epicycloid when the relation of the rolling to the stationary circle is* 1 : 2.

Solution.—Let A B and 6 A be the diameters of the given circles, having the proportion of 2 : 1, respectively. Divide the rolling circle into any number of equal parts (12), and as circumferences are proportional to diameters, the circumference of 6 A = the semi-circumference B A contains 12 of the same equal parts. With center C draw circles passing through points 1, 2, 3, 4, 5, 6 and D, and also the diameters D′a, D″b, D‴c, etc. D′, D″, D‴ are the centers and radius D A to arcs a A′, b A″, c A‴, etc.

Connect A, A′, A″, A‴, etc., by a curve, which is the required epicycloid.

135.—FIG. 2.—**Problem.**—*To construct a hypocycloid when the relation of the circles is as* 1 : 2.

Solution.—Treating this construction as the previous one, we shall obtain a right line A B as the required hypocycloid.

This construction is the fundamental principle of the *planet wheel*, applied to convert directly a rotation into a reciprocating movement (pump-piston).

136.—FIG. 3.—**Problem.**—*To construct a hypocycloid.*

Solution.—Let C be the circle, point A the generating point rolling in circle E. Relation of circles 1 : 3. Make an equal division in both circles (A b = A 1) and draw radii A C, b C′, c C″, etc. C, C′, C″, C‴, etc., are the centers and C A the radius to arcs b A′, c A″, d A‴, etc. Connect A, A′, A″, A‴ by a curve, which is the required hypocycloid.

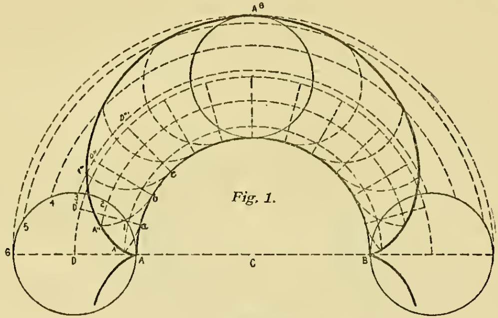


Fig. 1.

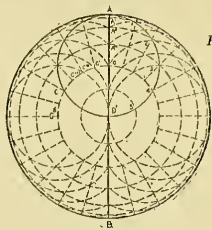


Fig. 2.

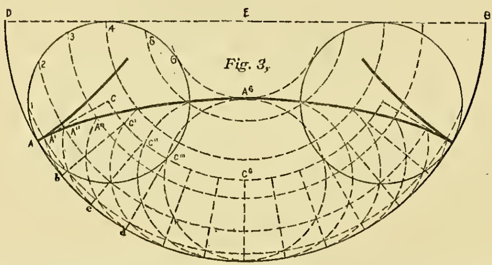


Fig. 3,

Hanstein's Constructive Drawing.

137.—FIG. 1.—**Problem.**—*To construct an epitrochoid.*

It will not be difficult to execute this curve. See Fig. 2, Plate 29.

138.—FIG. 2.—**Problem.**—*To construct a hypotrochoid.*

Solution.—Let C F be the rolling circle, A the generating point and D J B the circumference on which circle C rolls. Proceeding as in Figs. 2 and 3, we obtain the curve A, A′, A″, A‴, etc., which is the required hypotrochoid.

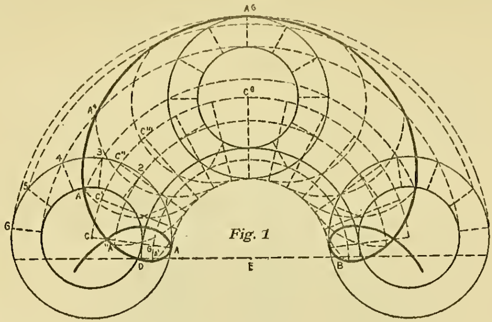


Fig. 1

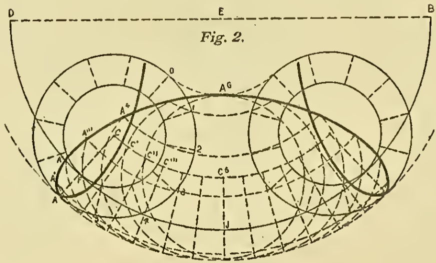


Fig. 2.

Hanstein's Constructive Drawing.

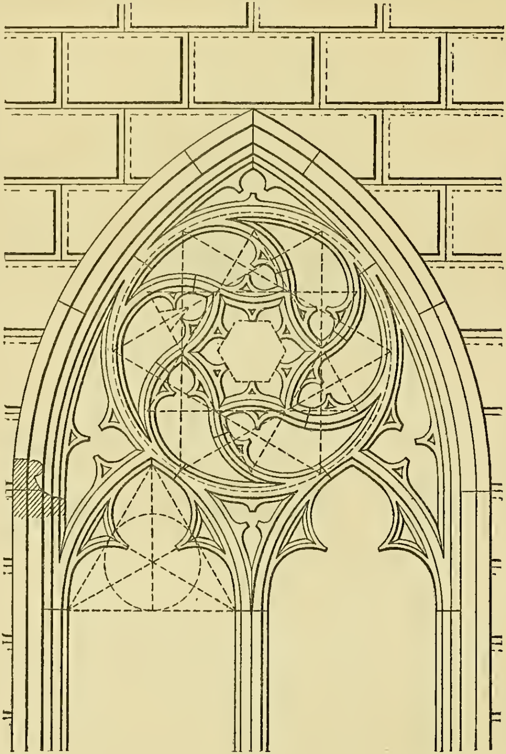
APPLICATIONS TO ARCHITECTURE.

139.—FIG. 1.—**Problem.**—*To construct a design for an ornamented Gothic arch in stone.*

This construction is based on principles explained and described in the previous part of this volume, and its solution should not present any serious obstruction to the student.

Remark.—To obtain an accurate result, it is advisable to make the equilateral triangle, the fundamental figure of this arch, not less than 8 inches a side.

Fig. 1.

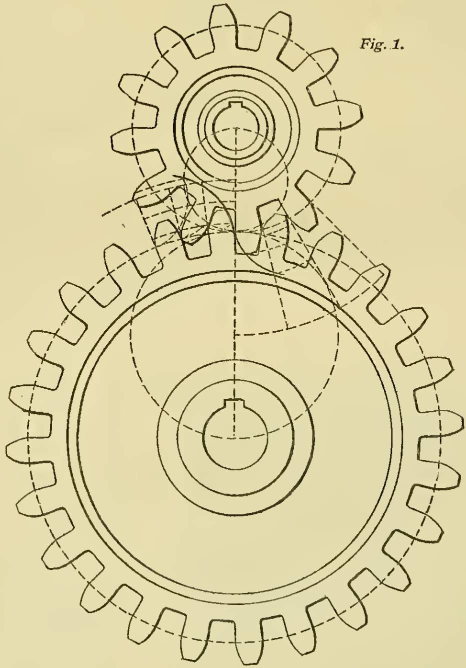


Hanstein's Constructive Drawing.

140.— FIG. 1.— **Problem.**— *To construct a pair of spur-wheels, their relation to be* 1 : 2.

To solve this problem we require the construction of two epicycloids and two hypocycloids to the "*flanks*" of the teeth, and it is advisable to enlist the advice of a teacher, to execute this important construction correctly.

Fig. 1.



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